Knowledge Organisers and Practice questions

Year 9 Autumn term test covers Topics 1 – 3
Year 9 Spring term test covers Topics 1 – 6
Year 9 Summer term test covers Topics 1 – 12
Year 10 Autumn term test covers Topics 1 – 16
Year 10 Spring term test covers Topics 1 – 19
Year 10 Summer term test covers Topics 1 – 23
Year 11 Autumn term Mock covers Topics 1 – 26

GCSE Higher Mathematics

Topic	Title		
1	Factors and Multiples	Written calculations: non calculator $+, -, \times, \div$	◯
		Negative numbers;	◯
		Prime factor decomposition;	◯
		HCF and LCM from Venn diagrams	◯
		Rounding and approximations	◯
		Bounds: error intervals and UB/LB of calculations	◯
2	Indices and Standard Form	Indices: positive, negative, fractional, negative fractions, reciprocals	◯
		Standard form;	◯
3	Expressions 1	Simplifying expressions;	◯
		Expanding and factorise single brackets;	◯
		Expanding and factorise double brackets: incl. difference of two squares	◯
		Factorise quadratics with $2x^2, 6x^2, 7x^2$ etc.	◯
4	Equations 1	Form and solve linear equations;	◯
		Inequalities: number lines, solving linear, double sided	◯
		Change the subject: single and where subject occurs twice	◯
5	Charts and Averages	Sampling: random, systematic and stratified	◯
		Capture – recapture	◯
		Bar and Pie charts	◯
		Scatter charts: lines of best fit, interpretations and extrapolations	◯
		Averages from lists;	◯
		Average from tables;	◯
		Reverse means;	◯
		Histograms	◯

GCSE Higher Mathematics

Topic	Title		
6	Area and Volume 1	Area: triangle, parallelogram, trapezium, circle, non-right-angled triangle	◯
		Arc length;	◯
		Area sector;	◯
		Area segment;	◯
		Volume of prisms	◯
		Surface area of prisms	◯
7	Fractions, Decimals and Percentages 1	Fractions: add, subtract, multiply and divide	◯
		Find fractions of an amount	◯
		Find percentages of an amount	◯
		Percentage change, compound interest and depreciation	◯
		Reverse percentages	◯
		FDP conversions	◯
8	Ratio	Splitting into a ratio problems;	◯
		Speed, density, pressure;	◯
		Direct and inverse proportion	◯
9	Shapes and Angles	Properties of quadrilaterals;	◯
		Interior angles sums of polygons; using exterior angles to find sides	◯
		Angles about a point, on a straight line and in a triangle;	◯
		Angles on parallel lines (corresponding, alternate and vertically opposite)	◯
10	Pythagoras	Pythagoras' Theorem to find sides;	◯
		Simplifying surds; Expanding brackets with surds;	◯
		Rationalising the denominator: single and double denominators	◯
11	Trigonometry	Trigonometry in right angled triangles to find angles;	◯
		Trigonometry in right angled triangles to find sides;	◯

GCSE Higher Mathematics

Topic	Title		
12	Sequences	nth term of linear sequences;	○
		nth term of quadratic sequences;	○
		nth term of Geometric sequences;	○
		Fibonacci sequences	○
		Iteration: changing subject; showing roots exist; finding next terms	○
13	Probability	Combinations;	○
		Sample space;	○
		Venn diagrams and set notation	○
		Tree diagrams: unconditional and conditional	○
14	Transformations	Translation;	○
		Rotation;	○
		Reflection;	○
		Enlargement;	○
		Plans and elevations	○
15	Cumulative Frequency	Drawing cumulative frequency curves	○
		Using CF to find median and IQR	○
		Boxplots: drawing, reading and comparing	○
16	Equations 2	Solving quadratic equations by factorising;	○
		Solving quadratic equations by the quadratic formula;	○
		Solving quadratic equations by completing the square	○
17	Area and Volume 2	Similarity: lengths, area, volume	○
		Volume of sphere, cone and frustum	○
		Volume incl. ‘melted down’ shapes	○
		Surface area	○
18	Graphs 1	Drawing straight lines;	○
		Give the gradient and equation of a straight line;	○
		Parallel and perpendicular lines;	○
		Midpoints and line length	○

GCSE Higher Mathematics

Topic	Title		
19	Constructions	Constructions: triangles, perpendicular bisectors, angle bisectors	○
		Loci;	○
20	Equations 3	Plotting quadratic graphs;	○
		Simultaneous linear equations;	○
		Simultaneous quadratic equations;	○
		Linear inequalities;	○
		Quadratic inequalities;	○
		Set notation	○
21	Circle Theorems	Circle theorems;	○
		Proof of circle theorems	○
22	Graphs 2	Recognising graphs: linear, quad., cubic, circle, exponential, reciprocal	○
		Equation of a circle;	○
		Translating graphs;	○
		Gradients of curves;	○
		Area under curves	○
23	Further Trigonometry	Sine and Cosine rules to find angles or sides;	○
		Trigonometric graphs;	○
		Transforming graphs;	○
		Exact values	○
24	Functions	Function notation;	○
		Composite functions;	○
		Inverse functions	○
25	Proof and Fractions	Algebraic fractions;	○
		Algebraic proof;	○
26	Vectors	Geometric proof;	○
		Column vectors	○
		Vectors around a shape	○
		Vector proof for parallel or collinear line segments	○

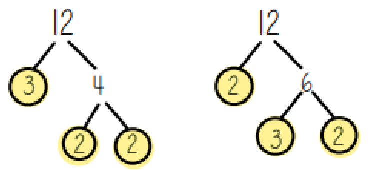
GCSE Higher Topic 1 Factors and Multiples Student Knowledge Organiser

Key words and definitions

Factor: a number that divides into another number exactly and without leaving a remainder.
Prime number: A prime number has only two factors - the number itself and 1. 1 is not a prime number
Multiple: This is the result of multiplying a number by an integer. The times tables of a number.
Product: the result when terms are multiplied together
Error Interval: the range of values a number could have taken before being rounded or truncated

Product of Prime Factors

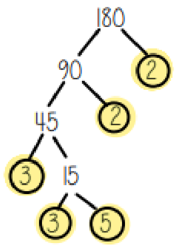
Example 1
Write 12 as a product of it's prime factors



Both of these trees represent the same decomposition

$12 = 2 \times 2 \times 3$
$= 2^2 \times 3$

Example 2
Write 180 as a product of it's prime factors



$180 = 2 \times 2 \times 5 \times 3 \times 3$
$= 2^2 \times 3^2 \times 5$

Always try to write your final answer in ascending order using index notation!

Using prime factor decomposition
If we know that 12 written as a product of it's prime factors, how does that help us to write 36 as a product of it's prime factors?

We know $12 \times 3 = 36$ therefore we can multiply our answer by three and
$36 = 2 \times 2 \times 3 \times 3$
$= 2^2 \times 3^2$

What about 120?
Well 120 is 10×12 so we can say
$120 = 2 \times 2 \times 3 \times 10$ Remember $10 = 2 \times 5$
$= 2^3 \times 3 \times 5$

Lowest Common Multiple

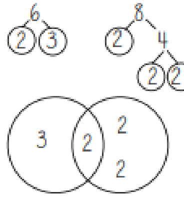
Example 1
What is the LCM of 6 and 8?

6 - 6, 12, 18, 24, 30
8 - 8, 16, 24, 32, 40

The first time their multiples match is 24 therefore:

the LCM of 6 and 8 is 24

Example 2
What is the LCM of 6 and 8?



We just multiply all of the numbers in the Venn diagram together to find the LCM

LCM of 6 and $8 = 3 \times 2 \times 2 \times 2$
$= 24$

Highest Common Factor

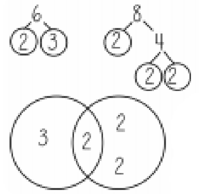
Example 1
What is the HCF of 6 and 8?

6 - 1, 2, 3, 6
8 - 1, 2, 4, 8

The biggest number which is a factor of both 6 and 8 is 2, therefore

the HCF of 6 and 8 is 2

Example 2
What is the HCF of 6 and 8?



As we are looking for the highest common factor we are looking for the factors which the two numbers share. These can be found in the overlap in the Venn diagram!

HCF of 6 and 8 = 2

Rounding – Decimal Places

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46
2.47

2.46 | 192

This shows the number is closer to 2.46

Rounding – Significant figures

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

We count significant figures from the first non-zero digit

Error Intervals

An error interval is a way of representing the upper and lower bounds of a value as an **inequality**.

Eg: w has been rounded to 6.4cm correct to one decimal place. Lower Bound = **6.35** Upper Bound = **6.45**

The error interval for w is: $6.35 \leq w < 6.45$

Calculations with Bounds

A = 30 to nearest whole number **LB = 29.5** **UB = 30.5**
B = 11.5 to 1 decimal place **LB = 11.45** **UB = 11.55**
C = 300 to 1 significant figure **LB = 250** **UB = 350**

Calculate the **maximum** value of **A + B**
UB of A + UB of B = 30.5 + 11.55 = 42.05

Calculate the **minimum** value of **A x C**
LB of A x LB of B = 29.5 x 250 = 7375

Calculate the **maximum** value of **C ÷ B**
UB of C ÷ LB of B = 350 ÷ 11.45 = 30.57 (2dp)

GCSE Higher Topic 1 Factors and Multiples Student Knowledge Organiser

Product of prime factors

Write the following as the product of their prime factors

(a) 70
(b) 90
(c) 24
(d) 126
(e) 75
(f) 84
(g) 99
(h) 500

HCF and LCM

By expressing the following numbers as products of their prime factors and then drawing a Venn diagram, can you find the HCF and LCM of:

(a) 12 and 28
(b) 28 and 42
(c) 48 and 64
(d) 15 and 25
(e) 12 and 32
(f) 30 and 105
(g) 28 and 126

(h) The lowest common multiple of two numbers is 36, one number is 12, what might the other number be?

(i) Jack thinks of two numbers, the HCF of these numbers is 6 and one of the numbers is 24 suggest what his other number may have been.

Rounding – decimal places and significant figures

Round the following numbers to the given number of decimal places:

(a) 4.763 (1dp)
(b) 0.543 (2dp)
(c) 12.895 (2dp)
(d) 2.956 (2dp)
(e) 7.895 (2dp)
(f) 1.998 (2dp)
(g) 1.005 (2dp)
(h) 0.0996 (3dp)

Round the following numbers to the given number of significant figures:

(a) 36.937 (3sf)
(b) 20643 (2sf)
(c) 19.6754 (4sf)
(d) 23139 (3sf)
(e) 258 (2sf)
(f) 0.04319 (2sf)
(g) 0.00348 (2sf)
(h) 7999032 (1sf)

By rounding all values in the calculation to 1 significant figure, **estimate** the answers to the following calculations:

(a) $\dfrac{480 \times 1.94}{4.7 \times 3.8}$

(b) $\dfrac{164.7 \times 4.2}{8.24 \times 2.09}$

(c) $\dfrac{5.79 \times 312}{0.523}$

(d) $\dfrac{29.8 \times 4.1}{0.21}$

Error Intervals

1. The number of passengers on a coach, g, rounded to the nearest 10 is 70 people. Write down the error interval for g

2. A number, g, rounded to the nearest whole number is 241. Write down the error interval for g

3. The density of an alloy, m, correct to 2 significant figures is 5.9g/cm^3. Write down the error interval for m

4. A number, p, **truncated** to 2 decimal places is 13.19. Write down the error interval for p

5. The weight of a pencil case, w, rounded to the nearest 100g is 900g. Write down the error interval for w

Bounds Calculations

Question 1

$$m = \frac{1}{ps}$$

p = 5.37 correct to 2 decimal places.
s = 2.9 correct to 1 decimal place.

Calculate the upper bound for the value for m. You must show your working.

Question 2

$$r = p + \frac{1}{q}$$

p = 4.3 correct to the nearest 0.1
q = 0.4 correct to the nearest 0.1

Work out the upper bound for r. You must show all your working.

Question 3

$$D = \frac{x}{y}$$

x = 99.7 correct to 1 decimal place.
y = 67 correct to 2 significant figures.

Calculate the lower bound for the value of D. You must show your working.

GCSE Higher Topic 2 Indices and standard form Student Knowledge Organiser

Key words and definitions

Base – The number that gets multiplied by a power
Index number - number that is multiplied by itself one or more times is raised to a power. The power is the index number. The plural is indices.
Standard Form – A system used to write both large and small numbers as a number between 1 and 10 multiplied by a power of 10
Integer – A whole number
Reciprocal – The reciprocal of a number is 1 divided by the number. A number multiplied by its reciprocal will always give an answer of 1.

Index Laws

Rule	
$a^m \times a^n = a^{m+n}$	$2^5 \times 2^3 = 2^8$
$a^m \div a^n = a^{m-n}$	$5^7 \div 5^3 = 5^4$
$(a^m)^n = a^{m \times n}$	$(10^3)^7 = 10^{21}$
$a^1 = a$	$17^1 = 17$
$a^0 = 1$	$34^0 = 1$

Negative Indices

Rule

$$a^{-m} = \frac{1}{a^m}$$

a^{-m} is the reciprocal of a^m

Examples

$$9^{-2} = \frac{1}{9^2} = \frac{1}{81}$$

$$\left(\frac{1}{3}\right)^{-3} = \left(\frac{3}{1}\right)^{3} = 27$$

Fractional Indices

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = \left(\sqrt[m]{a}\right)^n$$

$$25^{\frac{1}{2}} = \sqrt{25} = \underline{5}$$

$$25^{\frac{3}{2}} = \left(\sqrt[2]{25}\right)^3 = 5^3 = \underline{125}$$

$$8^{\frac{1}{3}} = \sqrt[3]{8} = \underline{2}$$

$$(81x^2)^{\frac{1}{2}} = \sqrt{81x^2} = \underline{9x}$$

Negative Fractional Indices

$$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \underline{\frac{1}{2}}$$

Remember this means the cube root of 8!

$$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{5^3} = \underline{\frac{1}{125}}$$

Think about what the negative in the power does first.

Standard Form Conversions

Numbers in standard form should always be written in the following format:

$$A \times 10^n$$

This power is always an integer

A is 1 or greater, but less than 10

Write 32 000 000 in standard form:

$32\,000\,000$
$= 3.2 \times 10\,000\,000$
$= 3.2 \times 10^7$

Write 8.35×10^{-3} as an ordinary number:

8.35×10^{-3}
$= 8.35 \times 0.001$
$= 0.008\,35$

Standard Form Calculations

Add and Subtract

$6 \times 10^5 + 8 \times 10^5$

The powers **must** be the same before we add or subtract

$= (6 + 8) \times 10^5$
$= 14 \times 10^5$
$= 1.4 \times 10^1 \times 10^5$

Be careful! This is **not** standard form!

$= \underline{1.4 \times 10^6}$

Multiply and Divide

For multiplication and division you can look at the values of A and the powers of 10 as separate calculations.

$$\frac{1.5 \times 10^5}{0.3 \times 10^3}$$

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$(1.5 \div 0.3) \times 10^5 \div 10^3$

$= \underline{5 \times 10^2}$

GCSE Higher Topic 2 Indices and standard form Student Knowledge Organiser

Index Laws

Simplify the following:

(a) $2^2 \times 2^2$ (b) $2^2 \times 2^3$ (c) $2^6 \times 2^2$

(d) $5^7 \div 5^5$ (e) $5^3 \div 5$ (f) $5^8 \div 5$

(g) $(8^6)^6$ (h) $(8^9)^2$ (i) $(8^4)^8$

Simplify the following using index laws:

$b^4 \times b^{12} \times b^2$

$a^2 \times b^3 \times a^{10} \times b^4$

$a(a^3 \times (a^6)^3)$

$(a^3)^3 \times (a^4)^2$

$b^2(b^8 \div (b^2)^2)$

$(d^5)^2 \div d^8$

$(a^5)^5 \div (a^{13})^2$

$4a \times 5a$

$3a^2 \times 5a^3$

$40b^2 \div 2b$

Negative Indices

Evaluate the following:

(a) 5^{-2} (b) 2^{-1} (c) 2^{-3}

(d) 4^{-2} (e) 3^{-3} (f) 6^{-1}

Write each of the following as fractions:

(a) a^{-2} (b) y^{-1} (c) w^{-4}

(d) 2^{-x} (e) 5^{-a} (f) x^{-n}

Fractional Indices

Evaluate the following:

(a) $25^{\frac{1}{2}}$ (b) $81^{\frac{1}{2}}$ (c) $4^{\frac{1}{2}}$

(d) $144^{\frac{1}{2}}$ (e) $8^{\frac{1}{3}}$ (f) $125^{\frac{1}{3}}$

(a) $8^{-\frac{2}{3}}$ (b) $25^{-\frac{3}{2}}$ (c) $64^{-\frac{2}{3}}$

Evaluate the following:

$\left(\frac{25}{36}\right)^{1/2}$ $\left(\frac{27}{125}\right)^{2/3}$ $25^{-1/2}$ $\left(\frac{8}{27}\right)^{-2/3}$

Converting Standard Form

Convert to standard form

(e) 100000000 (f) 900 (g) 250000

(i) 54000000 (j) 11000000 (k) 89000

(e) 0.00065 (f) 0.0022 (g) 0.0361

(i) 0.00000423 (j) 0.0000000981 (k) 0.00407

Convert from standard form

(e) 5×10^7 (f) 1.2×10^2 (g) 2.9×10^5

(i) 3.16×10^{-5} (j) 8.62×10^{-4} (k) 7.09×10^{-6}

Standard Form Calculations

Without using a calculator, work out the following:

(a) $3.57 \times 10^3 \times 6.7 \times 10^7$ (b) $9.5 \times 10^4 + 3.8 \times 10^5$

(c) $1.8 \times 10^9 \times 5.2 \times 10^9$ (d) $7 \times 10^{-8} \times 2 \times 10^{-6}$

(e) $(7.71 \times 10^{15}) \div (6 \times 10^4)$ (f) $(8 \times 10^9)^3$

(g) Write these numbers in order of size. Start with the smallest number.

2.5×10^2 0.0025 2.5×10^{-2} 2500

(h) Write these numbers in order of size. Start with the smallest number.

0.0034×10^5 34×10^{-5} -3.4×10^{-3} 3.4×10^4 34×10^2

GCSE Higher Topic 3 Expressions 1 Student Knowledge Organiser

Key words and definitions

Simplify – reduce an algebraic expression to its simplest terms

Expand – Multiply to remove the brackets from an expression

Factorise – Put brackets into an expression by identifying the common factors of the terms in the expression

Quadratic – An equation of expression where the unknown is raised to the power of 2 (it is squared)

Expanding and simplifying single brackets

Expand $-3(2x+4)$.

Solution

Multiply the **first term.**

$-3(2x+4)$

Multiply the **second term.**

$= -6x - 12$

Expand and simplify $x(x+4) - 3(x-2)$

$x(x+4) - 3(x-2)$

Expand as single brackets.

$= x^2 + 4x - 3x + 6$

$= x^2 + x + 6$

Simplify by collecting like terms

Factorising Single Brackets

Factorise $2ab + 4b$.

Find the **HCF** of the **variables.**

$2ab + 4b$

$= 2b(a+2)$

HCF $= 2$

HCF $= b$

Divide each term by the **HCFs** and **close the bracket.**

Only **'open the brackets'** once **all HCFs are found.**

Expanding Double Brackets

Outer

First

$(x+3)(x+4)$

Inner

Last

First
Outer
Inner
Last

$=$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

Expand Triple Brackets

Expand and simplify $(x+1)(x+2)(x+3)$

$(x+1)(x+2)(x+3)$

Expand the **first two brackets only.**

$= x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

$(x^2 + 3x + 2)(x+3)$

Expand the **result with the next bracket.**

$= x^3 + 3x^2 + 3x^2 + 9x + 2x + 6$

$= x^3 + 6x^2 + 11x + 6$

Factorise Quadratics

Factorise $x^2 + 9x + 18$

$x^2 + 9x + 18$

$= (x+6)(x+3)$

Write a **list** of **factor pairs** of the **constant term.**

Factors of 18

1, 18

2, 9

3, 6

Choose the pair that **sum to "+9".** These numbers go in the brackets

Factorising more challenging Quadratics

Factorise $5x^2 + 2x - 3$

Multiply the **coefficient of x^2** by the **constant term** & write a **list** of **factor pairs.**

$5x^2 + 2x - 3$

$\Rightarrow (5x+5)(5x-3)$

5

$= (x+5)(5x-3)$

$5 \times (-3) = -15$

Factors of 15

1, 15

3, 5

Choose the pair that can be used to make "+2".

Set up the **double brackets** using the **coefficient of x^2.**

$5x \times 5x = 25x^2$ so we need to **divide** the **brackets by their HCFs to get our final answer**

The difference of two squares

$$a^2 - b^2 = (a+b)(a-b)$$

Factorise $x^2 - 16$

$= x^2 - 4^2$ $\quad 16 = 4^2$

$= (x+4)(x-4)$

Factorise $4x^2 - 81y^2$

$= (2x)^2 - (9y)^2$

$= (2x+9y)(2x-9y)$

GCSE Higher Topic 3 Expressions 1 Student Knowledge Organiser

Expanding and factorising Single Brackets

Expand and simplify where possible:

[a] $a(a+6)$

[b] $r(2r+7)$

[c] $s(3s-1)$

[d] $-h(h+5)$

[a] $5(x+6)-3(x-1)$

[b] $7(x-5)+6(x+2)$

[c] $2(x+1)-2(x-1)$

[d] $3(x+3)+(x-8)$

Fully factorise the following expressions

[a] $3d+12$

[b] $6-9h$

[c] $12-18e$

[d] $14+35r$

[e] $2x^2y^2+6xy^2$

[f] $14u^3t-21u^2t$

[g] $9x^2+3x-6xy^2$

[h] $4p^2q-6pq^3+2pq$

Expanding Double Brackets

Expand and simplify the following:

[a] $(x+4)(x+1)$ **[b]** $(x-9)(x+2)$

[c] $(x+3)(x+6)$ **[d]** $(x-3)(x-2)$

[e] $(x+7)(x-2)$ **[f]** $(x+5)(x-10)$

Expand and simplify the following:

[a] $(3x-2)(x+6)$ **[c]** $(2x-1)(3x+2)$

[b] $(4x+3)(2x-5)$ **[d]** $(5x-3)^2$

Expanding Triple Brackets

Expand the following sets of triple brackets

[a] $(x+1)(x-2)(x-3)$

[b] $(x-1)(x+2)(x+3)$

[c] $(x+1)(x-2)(x+3)$

[d] $(x-1)(x+2)(x-3)$

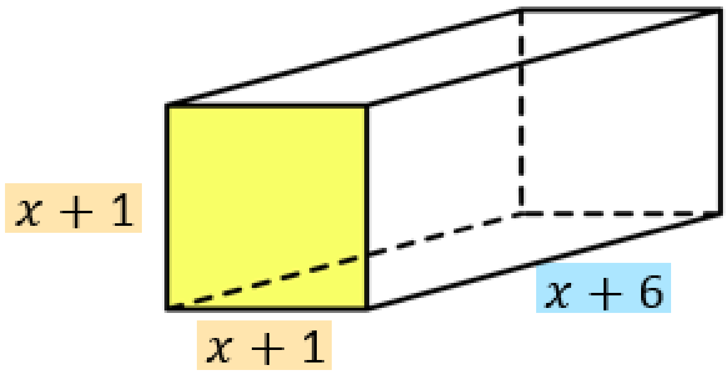
[f] $(x+1)(x-1)^2$

[g] $(x-4)^2(x+2)$

[h] $(x+1)^2(x-5)$

[i] $(x+2)^3$

Write a simplified expression for the volume of the following cuboid.



Factorising Quadratics

Factorise the following expressions:

[a] x^2-5x+6

[b] $x^2+14x+48$

[c] $x^2-8x+12$

[d] $x^2+19x+88$

[e] $x^2-21x+110$

[f] x^2+2x+1

[g] $x^2+14x+24$

[h] $x^2+15x+56$

[i] $x^2-13x+40$

[j] $x^2-17x+42$

[k] $x^2+4x+16$

[l] $x^2-16x+15$

[m] $x^2+20x+75$

[n] $x^2+23x+120$

[o] $x^2-20x+96$

[p] $x^2+17x+52$

Factorising Harder Quadratics

Factorise the following expressions:

[a] $6x^2-13x+5$

[b] $12x^2-7x+1$

[c] $9x^2-9x-4$

[d] $6x^2+7x-3$

[e] $9x^2+15x+4$

[f] $12x^2+13x-4$

GCSE Higher Topic 4 Equations 1 Student Knowledge Organiser

Key words and definitions

Equation: An equation says that two things are equal
Expression: Is a set of terms combined using the operations
Variable: A symbol (usually a letter) standing in for an unknown value
Linear: Linear functions are those whose graph is a straight line
Subject: The variable that is being worked out
Inequality: Compares two values, showing if one is less than, greater than, or simply not equal to another value
Integer: Whole number

Solving 2-Step Linear Equations

$$\begin{aligned} 8b + 5 &= 29 \\ -5 \quad & \quad -5 \\ 8b &= 24 \\ \tfrac{8b}{8} &= \tfrac{24}{8} \\ \boxed{x = 3} \end{aligned}$$

Step 1: Subtract 5 from both sides.
Step 2: Divide both sides by 8.

Adding Integers
Solve One-Step Equations
Dividing Integers

$$\begin{aligned} -3x + (-2) &= -17 \\ +2 \quad & \quad +2 \\ -3x &= -15 \\ \div(-3) \quad & \quad \div(-3) \\ x &= 5 \end{aligned}$$

Solving Equations using Cross-Multiplication

Ex1 Solve,

Solution

$$\begin{aligned} \frac{3x-1}{x+2} &= 2 \\ \times(x+2) \quad & \quad \times(x+2) \\ 3x - 1 &= 2(x+2) \\ & \text{Expand} \\ 3x - 1 &= 2x \\ -2x \quad & \quad -2x \\ x - 1 &= 4 \\ +1 \quad & \quad +1 \\ x &= 5 \end{aligned}$$

Multiply to remove the denominator.

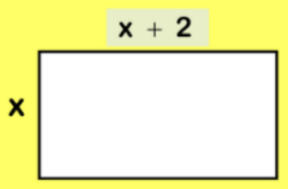
Ex3 Solve,

Solution

$$\begin{aligned} \frac{4x+2}{2x-3} &= 10 \\ \times(2x-3) \quad & \quad \times(2x-3) \\ 4x + 2 &= 10(2x-3) \\ & \text{Expand} \\ 4x + 2 &= 20x \\ -4x \quad & \quad -4x \\ 2 &= 16x - 30 \\ +30 \quad & \quad +30 \\ \frac{32}{16} &= \frac{\cancel{16}x}{\cancel{16}} \\ 2 &= x \end{aligned}$$

Multiply to remove the denominator.

Forming Equations



Perimeter:

$x + 2 + x + 2 + x + x$

$= 4x + 4$

Area:

$x + 2 \times x$

$= x^2 + 2x$

The sum of (x + 2), (x + 3) and (x + 4) is 21.
Work out x.

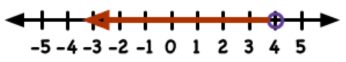
$$x + 2 + x + 3 + x + 4 = 21$$
$$3x + 9 = 21$$
$$3x = 12$$
$$x = 4$$

Inequalities

Greater than $>$
Greater than or equal to $\geq$
Less than $<$
Less than or equal to $\leq$
Not equal to $\neq$

Inequalities on a Number Line

WRITE AN INEQUALITY:

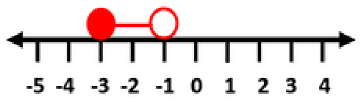


CIRCLES:
closed = included
open = not included

ARROWS:
left point = <
right point = >

$n < 4$

$-3 \leq x < -1$



Solving Inequalities

Solving Two-Step Inequalities

1. Add or subtract to isolate the variable term.
2. Multiply or divide to solve for the variable. If multiply or divide by a negative number then reverse the inequality symbol.

Example:

$$\begin{aligned} -3x + 5 &\leq -16 \\ -5 \quad & \quad -5 \qquad \text{Subtract} \\ -3x &\leq -21 \\ \frac{-3x}{-3} &\geq \frac{-21}{-3} \qquad \text{Divide by -3, reverse inequality} \\ x &\geq 7 \end{aligned}$$

Changing the Subject of the Formula

Make 'x' the subject of this formula $y = 3x + 5$

$y = 3x + 5$ (-5) (Subtract 5 from both sides)

$y - 5 = 3x$ $(\div 3)$ (Divide both sides by 3)

$\frac{y-5}{3} = x$ or $x = \frac{y-5}{3}$

Make c the subject

$A = 3b + 3c$ — Expand

$A - 3b = 3c$ — Subtract 3b

$\frac{A-3b}{3} = \frac{3c}{3}$ — Divide by 3

$\frac{A-3b}{3} = \frac{3c}{3}$ — Cancel out on the RHS

$\frac{A-3b}{3} = c$

GCSE Higher Topic 4 Equations 1 Student Knowledge Organiser

Solving Equations

Solve each of the following equations.

a) $8x + 10 = 66$ **b)** $10x + 15 = 115$ **c)** $12x + 9 = 105$ **d)** $15x + 12 = 72$
e) $1.5x - 3 = -24$ **f)** $1.8x - 8 = -62$ **g)** $2.6x - 7 = -59$ **h)** $4.8x - 9 = -57$

a) $7(x + 4) = 63$ **b)** $8(x + 4) = 88$ **c)** $11(x + 3) = 132$ **d)** $14(x + 5) = 98$
e) $16(x - 3) = -80$ **f)** $13(x - 4) = -91$ **g)** $14(x - 2) = -98$ **h)** $18(x - 3) = -180$

a) $6x - 4 = 2x + 16$ **b)** $17x - 2 = 7x + 8$ **c)** $9x - 26 = 5x - 14$
d) $10x - 5 = 3x + 9$ **e)** $6x - 12 = 51 - 3x$ **f)** $5x - 13 = 87 - 5x$

Solving Equations using Cross Multiplication

[a] $\dfrac{2x+3}{x-1} = 7$ [a] $\dfrac{2x-2}{2x+3} = 6$

[b] $\dfrac{4x-2}{x+5} = 2$ [b] $\dfrac{4x-1}{3x+1} = 1$

[c] $\dfrac{5x-3}{x-2} = 4$ [c] $\dfrac{6x-2}{4x+5} = 2$

[d] $\dfrac{3x-3}{x+3} = 6$ [d] $\dfrac{3x-3}{2x+2} = 1$

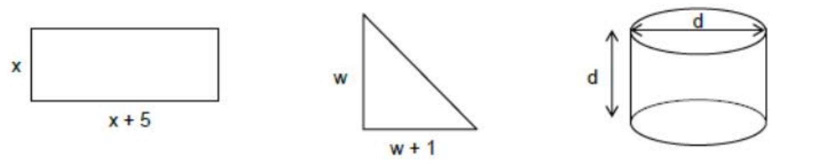
Forming Equations

1) *The length of a rectangle is twice its width, x. If the perimeter is 42 cm find the area of the rectangle.*

2) *The sum of four consecutive numbers is 90. Let x be the first number. Find the numbers.*

3) *Mushood buys x books at £5.50 each, and (x + 2) books at £3.50 each. The total cost of the books is £25. Find the value of x.*

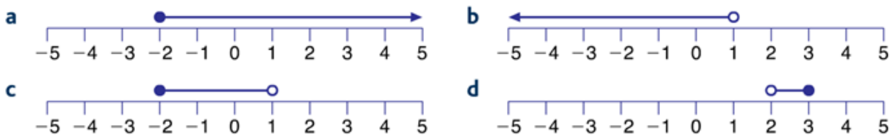
The area of this rectangle is 300 cm² | The area of this triangle is 6 cm² | The surface area of this cylinder is 150π cm²



1) Work out the value of x.
2) Work out the perimeter.
3) Work out the value of w.
4) Work out the length of the hypotenuse to 2 dp.
5) Work out the value of d.
6) Find the volume of the cylinder in terms of π.

Writing Inequalities on a Number Line

1 Write down the inequality shown on the number line:



2 Show these inequalities on a number line.

a $x > 2$ **b** $x < 5$ **c** $x \geqslant 0$ **d** $x \leqslant -1$

4 Show these inequalities on a number line.

a $-1 < x \leqslant 3$ **b** $-4 \leqslant x < 0$ **c** $-5 < x \leqslant -2$

Solving Inequalities

(a) $2x + 1 \leq 9$ (b) $3x - 5 > 16$ (c) $4x + 8 < 32$ (d) $5x - 2 \geq 68$

(e) $\dfrac{x}{2} + 1 \leq 5$ (f) $\dfrac{x}{9} - 6 > 4$ (g) $\dfrac{x+3}{2} \geq 5$ (h) $\dfrac{x-5}{4} > 2$

(a) $4x + 7 < 11$ (b) $3x - 2 \geq 10$ (c) $\dfrac{x}{2} - 3 > 0$ (d) $\dfrac{x+18}{4} \leq 5$

(a) $4x + 3 > 2x + 11$ (b) $x + 1 \geq 3x - 18$

(c) $13x - 12 < 3x + 13$ (d) $7x - 5 \geq 3x + 11$

Changing the Subject of the Formula

Question 2: Make x the subject of the following formulae

(a) $4x + c = w$ (b) $dx - t = 8$ (c) $x^2 + 3 = h$

(d) $2x + 2y = P$ (e) $s = x^2 - 3$ (f) $y = xz + s$

(g) $\dfrac{x}{n} + 2 = w$ (h) $\dfrac{x}{6} - 5 = w$ (i) $\dfrac{x+3}{c} = h$

(j) $3y = 4x + 1$ (k) $x^2 + a = v$ (l) $x^3 - 4 = 5y$

Question 1: Make x the subject of each of the following

(a) $A = \tfrac{1}{2}(x + y)$ (b) $A = \pi r^2 + 2\pi rx$

(c) $T = 3x^2 - y$ (d) $s = \dfrac{m}{ax}$

(e) $s = uy + \tfrac{1}{2}xy^2$ (f) $\tfrac{1}{3}w = \tfrac{1}{4}x + t$

Question 2: Make m the subject of the following formulae

(a) $5(m + y) = 4(m - 3y)$ (b) $3(3m + 4) = 7(m + 2a)$

(c) $15(2m + 3c) = 5(m + 7c)$ (d) $9m + 4c = 2(a + 3m)$

(e) $a(c + m) = 2(c + 3m)$ (f) $w(m + n) = x(m - n)$

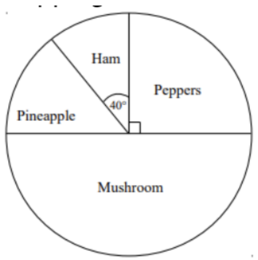
GCSE Higher Topic 5 Charts and averages Student Knowledge Organiser

Key words and definitions

Qualitative Data: Data which is non numeric
Quantitative Data: Data which is numeric
Discrete Data:
Continuous Data:
Mean: A type of average where all the data is added and divided by the amount of data
Mode: An average which is the most popular piece of data
Median: An average found when all data is put in order and middle value selected.
Range: Difference between the largest value and the smallest value

Pie Chart.

When drawing a pie chart, divide 360 by the total frequency. This will give you how many degrees are required for each part of the data. For example, in a survey of 40 people, if you do 360 divided by 40, it would mean each person would be represented by 9 degrees



Averages

Mean - Add up the values and divide by how many values there are.
Median - The middle value. Put the data in order and find the middle one
Mode - Most frequent/common.
Range - The difference between the highest and lowest vales

The mean of 3, 4, 7, 6, 0, 4, 6 is

$$\frac{3+4+7+6+0+4+6}{7} = 5$$

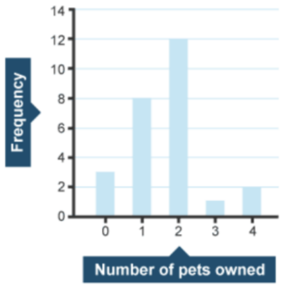
Find the median of:
4, 5, 2, 3, 6, 7,6
Ordered: 2, 3, 4, **5**, 6, 6, 7
Median = 5

Find the mode:
4, 5, 2, 3, 6, 4, 7, 8, 4
Mode = 4

Find the range:
3, 31, 26, 102, 37, 97
Range = 102-3 = 99

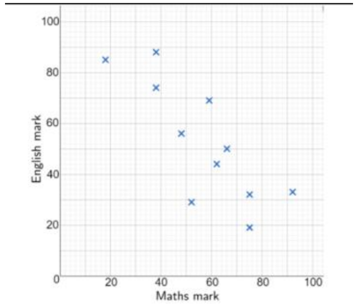
Bar Chart.

Represents data as vertical blocks. Each bar should be the same width. There should be gaps between each bar. Label each axis

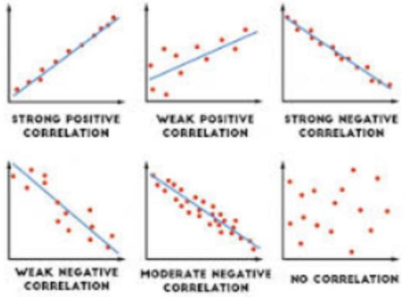


Scatter Graphs.

A scatter graph is used to plot data measured in two ways. Each point plotted is a single piece of data with two measurements.
Eg, each point on the following is for a single pupil, with their Maths and English scores from a test



When the points plotted on a scatter graph are all very close together, we say there is a strong **correlation** between the two things being measured. This might mean the two things are connected



Averages From Tables

You can use frequency tables to work out the Mead, Median, Mode and Range of a set of Data.

Pets	Frequency
0	12
1	7
2	11

Mode – The mode is the value with the highest frequency – 0
Median - Divide the total frequency by 2 to work out the middle value.
12+7+11=30, 30÷2=15
The first 12 values are 0. The next 7 values (so the 8th, 9th, 10th…) are 1.
So the median is the 15th value, which is 1.
Mean - Total = 0x12 + 1x7 + 2x11 =29
Total Frequency = 30
Mean= 29 ÷ 30 = 0.96666…

GCSE Higher Topic 5 Charts and averages Student Knowledge Organiser

Bar Charts

Question 1: The bar chart shows information about the hair colour of students in 7C.

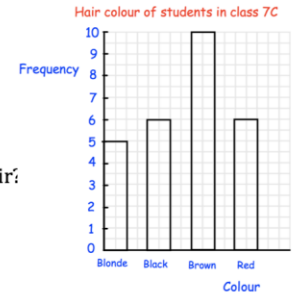
(a) What is the most common hair colour in 7C?

(b) How many students had black hair?

(c) What hair colour is the least popular in 7C?

(d) How many more students had brown than red hair?

(e) How many students are in 7C?



Question 1: Matthew is a milkman.
The table below shows information about how many pints of milk he delivers in one village.

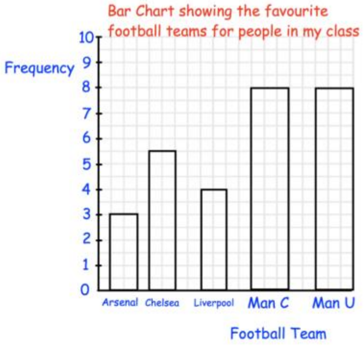
Day	Mon	Tues	Wed	Thurs	Fri	Sat
Pints Delivered	65	40	60	45	70	25

(a) Draw a bar chart to represent this information.
(b) How many pints of milk did he deliver in total?

Question 2: Shannon has drawn a bar chart to show the favourite football teams of the people in her class.

Shannon has made some mistakes.
(a) Explain what her mistakes are.
(b) Draw a correct bar chart for this information

Football Team	Frequency
Arsenal	3
Chelsea	5
Liverpool	4
Man City	8
Man United	8



Pie Chart

Question 5: 90 students went on a school trip to Longleaf Safari Park.
They were asked their favourite animals.
The pie chart shows the results.

(a) What fraction of the students chose elephant?

(b) What fraction of the students chose tiger?

(c) What fraction of the students chose giraffe?

(d) What fraction of the students chose rhino?

(e) Find x

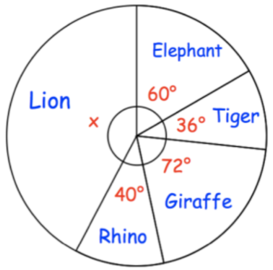
(f) How many students chose elephant?

(g) How many students chose tiger?

(h) How many students chose giraffe?

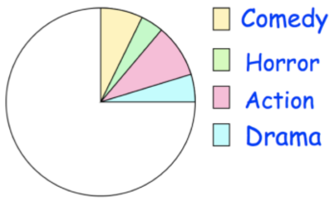
(i) How many students chose rhino?

(j) How many students chose lion?



Question 2: Bill has drawn a pie chart to show his friends' favourite genre of film.

Genre	Frequency
Comedy	26
Horror	14
Action	33
Drama	17



(a) Can you explain to Bill what he has done wrong?
(b) Draw a correct pie chart for Bill.

Scatter Graphs

Question 1: Plot the following information as scatter graphs

(a)

Maths score	9	13	6	18	11	4	15	10
Physics score	10	13	5	20	8	5	12	14

(b)

Age, years	4	7	2	4	1	9	3	6
Cost, £	6000	3000	7500	5000	8000	1500	6000	4000

(c)

Height, cm	157	160	148	160	177	156	166	170
Weight, kg	53	60	44	53	54	60	54	70

(d)

Distance, miles	2.5	0.8	1.2	4.1	2.8	3.3	3.7	1.5
Cost	£3.20	£1.40	£1.80	£4.40	£3.00	£3.60	£4.80	£2.40

Averages from tables

For each set of data, calculate the mean, mode and median

(a)

Age	Frequency
5	2
6	2
7	5
8	1

(b)

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

(c)

Number of pets	Frequency
0	13
1	28
2	50
3	9

Averages

For each data set, calculate the mode median, mean and range

(a) 5, 6, 6, 7, 8, 10

(b) 1, 1, 1, 4, 6, 8, 12

(c) 5, 5, 7, 7, 7, 8, 8, 9

(d) 5, 7, 3, 5, 8, 9, 10, 2

(e) 8, 3, 3, 4, 6, 8, 13, 3, 18

(f) 12, 14, 15, 17, 15

(g) 2.3, 2.6, 2.8, 2.7, 2.8, 2.7, 2.4, 2.3, 2.1, 2.3

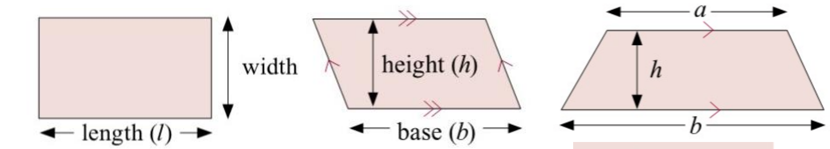
(h) −2, −1, 5, 8, −2, 2, −1, 9, −1, 1, 2, −1

GCSE Higher Topic 6 Area and volume 1 Knowledge Organiser

Key words and definitions

Perimeter: total distance around the edge of a shape
Perpendicular: two straight lines at right-angles to each other
Radius: distance from the centre to outer edge of a circle – notation is r
Diameter: distance from one side of a circle to the other passing through the centre – notation is d
Circumference: total distance around a circle
Arc: part of the circumference
Sector: part of a circle, cut from the centre to the edge (a pizza slice)
Π: Pi – mathematical value used when calculating with circles/curved shapes
Prism: 3D shape with constant cross-section through the entire length

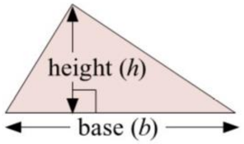
Area



Area = length × width

$A = bh$

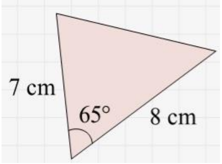
$A = \frac{1}{2}(a + b) \times h$



Area = $\frac{1}{2}$ × base × perpendicular height

You can also find the area of a triangle using Sine. You must know 2 sides and the angle formed between them.

$Area = \frac{1}{2}ab \sin C$

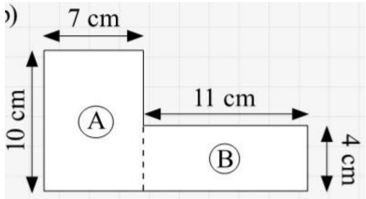


Area = $(\frac{1}{2} \times 7 \times 8)\sin 65°$

= **25.38 cm²** (to 2 d.p.)

Compound shapes – formed by merging multiple shapes

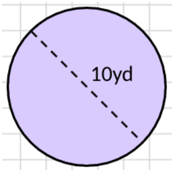
Split the shape up into basic shapes. Find the area of each, then add together.



Area of rectangle A	$= 10 \times 7$
	$= 70$ cm²
Area of rectangle B	$= 11 \times 4$
	$= 44$ cm²
Total area of shape	$= 70 + 44$
	$= \mathbf{114}$ **cm²**

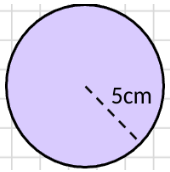
Circles – Circumference & Area

$C = \pi d$



$C = \pi d$
$= \pi \times 10$yd
$= 10\pi$ yd
$= 31.4$yd

$A = \pi r^2$



$A = \pi r^2$
$= \pi \times 5^2$
$= \pi \times 25$
$= 78.5$cm²

Sectors

Length of arc $= \frac{\theta}{360} \times$ **circumference of circle** $= \frac{\theta}{360} \times 2\pi r$

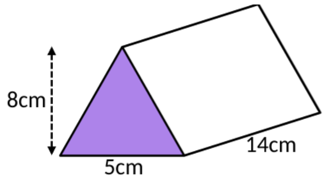
Area of sector $= \frac{\theta}{360} \times$ **area of circle** $= \frac{\theta}{360} \times \pi r^2$

Area of sector $= \frac{60}{360} \times \pi \times 3^2 = \frac{1}{6} \times 9\pi = \frac{3}{2}\pi$ **cm²**

Length of arc $= \frac{60}{360} \times (2 \times \pi \times 3) = \frac{1}{6} \times 6\pi = \pi$ **cm**

Volume of Prisms – example shown of triangular prism

Volume of a prism = area of cross section x length of prism



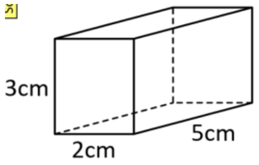
This volume formula works for all prisms. Only the formula for the cross-section area will change dependent on the shape.

Area of Δ $= \frac{5 \times 8}{2}$
$=$ **20cm²**

Volume $=$ **20cm²** $\times 14$
$=$ **280cm³**

Surface area of Prisms – examples of cuboid & cylinder

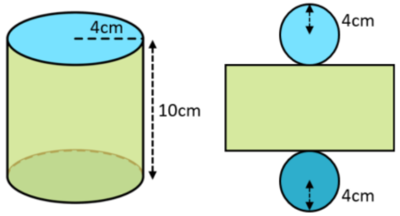
Surface area of a prism = sum of the areas of all the faces



Total surface area = 62 cm²

Front = 2 x 3 = 6
Back = 2 x 3 = 6
Top = 2 x 5 = 10
Base = 2 x 5 = 10
Left side = 3 x 5 = 15
Right side = 3 x 5 = 15

Surface area of cylinder $= 2\pi r^2 + \pi dh$ *special case



The 2 dimensions of the rectangular face are the circumference of the circular end and the height. So, the area of this face is π x d x h.

Area of ○ $= \pi r^2$
$= \pi \times 4^2 =$ **16π**

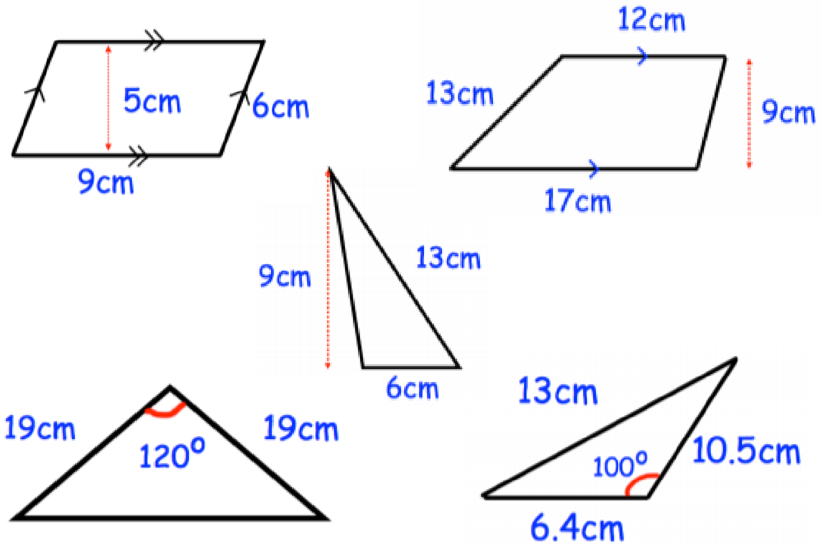
○ + ● = **16π** × 2 = **32π**

Area of ▭ $= 8\pi \times 10$
$=$ **80π**

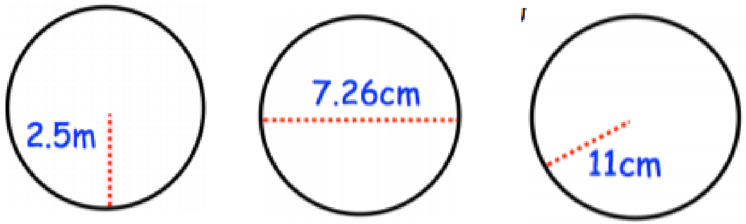
Total SA $= 32\pi + 80\pi$
$=$ **112π**cm²

GCSE Higher Topic 6 Area and Volume 1 Knowledge Organiser

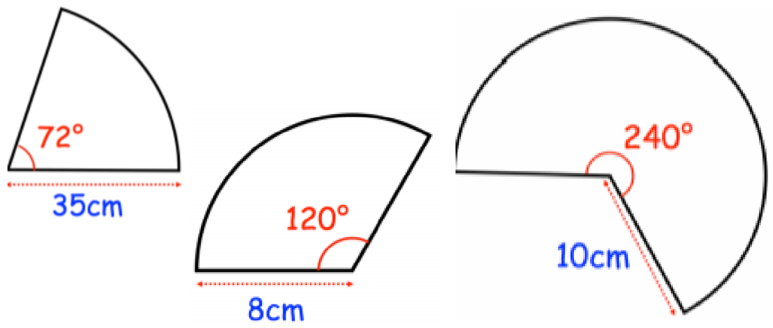
Area – Find the area of each shape



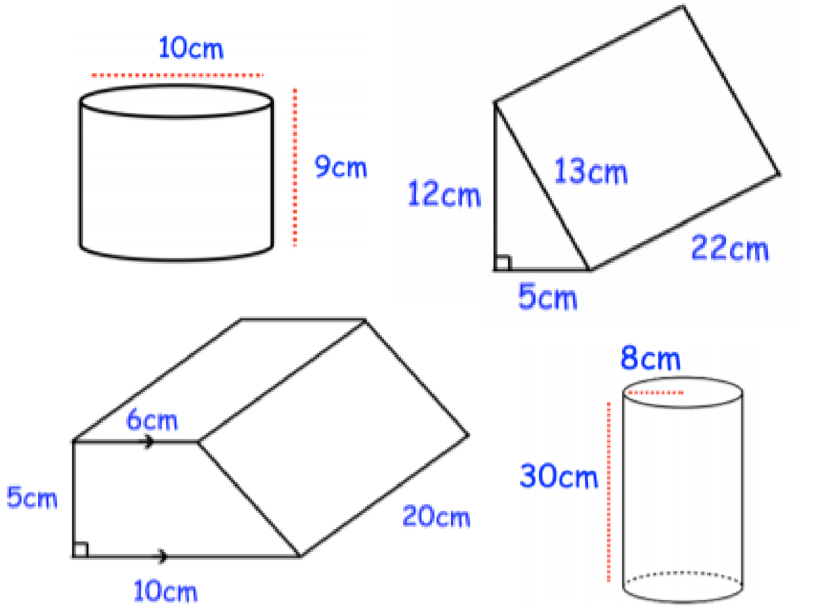
Circles – Find the area and circumference of each circle



Sector – Find the area and arc length of each sector

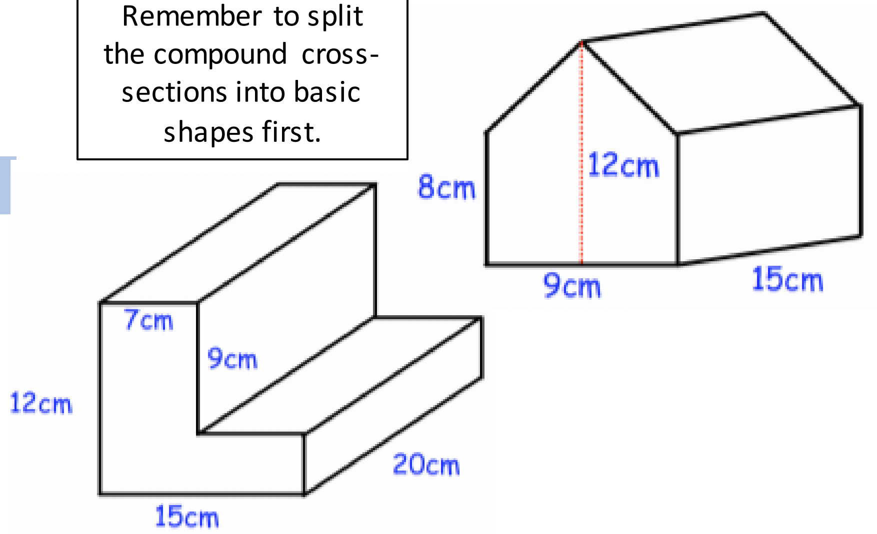


Volume – Find the volume of each 3D shape

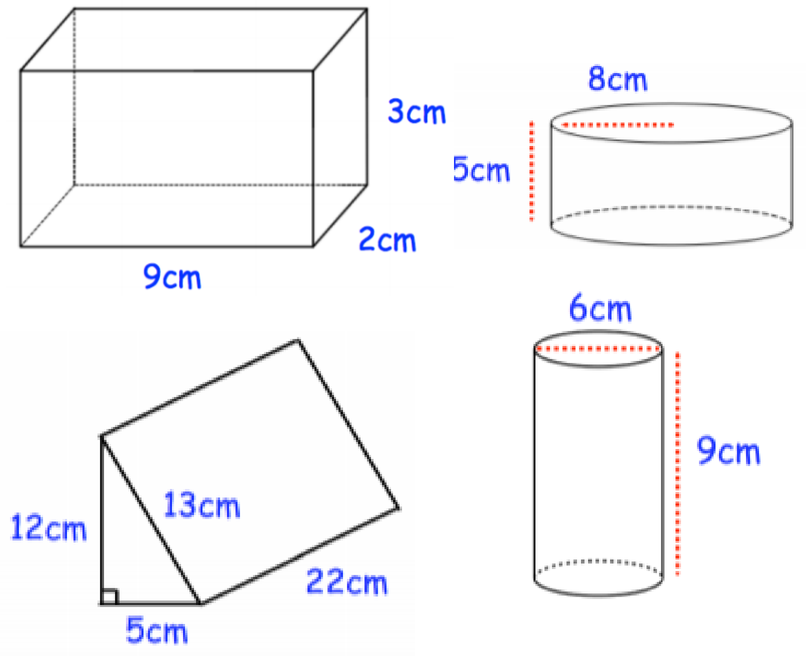


Volume – Find the volume of each compound 3D shape

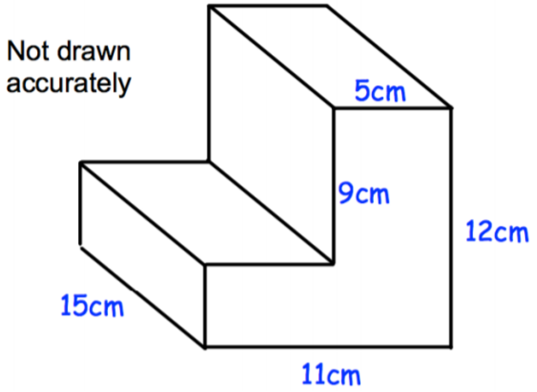
Remember to split the compound cross-sections into basic shapes first.



Surface Area – Find the surface area of each 3D shape



Surface Area – Find the surface area of the compound 3D shape



GCSE Higher Topic 7 Fractions, decimals and percentages 1 Student Knowledge Organiser

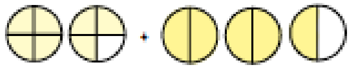
Key words and definitions

- Reciprocal – The reciprocal of a number is 1 divided by the number
- Simple Interest – Interest calculated as a percentage of the original amount
- Compound Interest – Interest calculated on the amount borrowed plus previous interest
- Equivalent – Of equal value
- Recurring Decimal – A decimal number with a digit, or group of digits, that repeat forever

Adding and Subtracting Mixed Numbers

Method 1

$1\frac{3}{4} + 2\frac{1}{2}$



We have three 'wholes', $\frac{3}{4} + \frac{1}{2}$

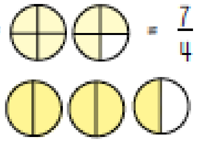
$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$

So we have:

$3 + 1\frac{1}{4} = 4\frac{1}{4}$

Method 2

$1\frac{3}{4} + 2\frac{1}{2}$

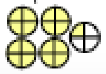


$1\frac{3}{4} = \frac{7}{4}$

$2\frac{1}{2} = \frac{5}{2} = \frac{10}{4}$

$\frac{7}{4} + \frac{10}{4} = \frac{17}{4} = 4\frac{1}{4}$

How many times does 4 go into 17? 4, 8, 12, 16, 20 — 4 with a remainder of 1



Multiplying and Dividing Fractions

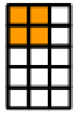
Multiplying any fractions

See cross-cancelling for a quicker method

Example 1

$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

"Two thirds of two fifths"



Another way to think of it: Two parts out of three parts on two out of five rows

Example 2

$\frac{5}{7} \times \frac{14}{15} = \frac{5 \times 14}{7 \times 15} = \frac{70}{105} = \frac{2}{3}$

Remember to simplify where possible!

Example 3

$1\frac{1}{2} \times 2\frac{1}{3}$

$\frac{3}{2} \times \frac{7}{3} = \frac{21}{6} = \frac{7}{2} = 3\frac{1}{2}$

Dividing Fractions

Example 1

$\frac{2}{3} \div \frac{5}{7}$

$\frac{2}{3} \times \frac{7}{5}$

$\frac{2 \times 7}{3 \times 5}$

$\frac{14}{15}$

Percentages of Amounts

Find 30% of 240

100% of 240 = 240

÷10: 10% of 240 = 24

×3: 30% of 240 = $\underline{72}$

A bar model to help visualise it:

24 x 3 = $\underline{72}$

Finding 10% is always a good place to start!

Find 81% of 480

100% of 480 = 480
10% of 480 = 48
1% of 480 = $\underline{4.8}$

100% of 480 = 480
10% of 480 = 48
80% of 480 = $\underline{384}$

80% + 1% = 81% so we need to add 4.8 and 384

81% of 480 = $\underline{388.8}$

Reverse Percentages

60% of a number is 48. What is the number?

60% of x = 48
10% of x = 8
100% of x = $\underline{80}$

A pair of shoes are on sale for 87.5% off. The sale price is £49.50, how much did they cost originally?

12.5% of x = £49.50
25% of x = £99
100% of x = $\underline{£396}$

- Compound Interest: 94
- Percentage Change: 97
- Reverse Percentages: 96

Compound Growth and Decay

I put £1000 in a bank account. It earns compound interest of 10% per year. How much will be in the account after 5 years?

INTEREST:
Compound interest means we work out the interest each year and the original amount plus any interest in the account.

- 10% of £1000 = £100

So after year 1, the account will have £1100

- 10% of £1100 = £110

So after year 2, the amount is £1210 etc…

If we are increasing by 10% each time, this is the same as finding 110% of the amount, or multiplying by 1.1 (see multipliers). So another way we can work this out is:

£1000 x 1.1 x 1.1 x 1.1 x 1.1 x 1.1

5 years

Or £1000 x 1.1^5 = $\underline{£1610.51}$

I know which account I'd go for!

For compound decay or depreciation questions we would do the same thing, just our multiplier at the start is calculated by subtracting rather than adding

Recurring decimals to fractions

Example (TWO RECURRING DIGITS)

Convert $0.\dot{3}\dot{5}$ to a fraction

x = 0.353535…
100x = 35.353535…

Because we have two digits that are repeating, we need to multiply it by 100!

99x = 35 → $x = \frac{35}{99}$

Example

Convert $0.2\dot{5}$ to a fraction

x = 0.255555…
10x = 2.5555555…
100x = 25.55555

Here, we cannot just take 2.555 away from 0.255 as we will not reduce it to an integer

25.555 - 2.55 = 23

100x - 10x = 90x

90x = 23 → $x = \frac{23}{90}$

GCSE Higher Topic 7 Fractions, decimals and percentages 1 Student Knowledge Organiser

Adding and Subtracting Fractions

Work out the following. Answers should be simplified and written as mixed numbers where necessary

(a) $\frac{3}{4}+\frac{1}{2}$ (b) $\frac{5}{9}+\frac{2}{3}$ (c) $\frac{7}{10}+\frac{1}{3}$

(d) $\frac{4}{5}-\frac{2}{3}$ (e) $\frac{8}{9}-\frac{1}{3}$ (f) $\frac{2}{3}+\frac{1}{6}$

Work out the following. Answers should be simplified and written as mixed numbers where necessary

(a) $1\frac{1}{2}+\frac{2}{3}$ (b) $\frac{7}{9}+1\frac{1}{3}$ (c) $1\frac{3}{5}-\frac{3}{4}$

(d) $1\frac{5}{8}-1\frac{1}{4}$ (e) $2\frac{1}{2}+1\frac{1}{3}$ (f) $2\frac{2}{9}-1\frac{1}{3}$

Multiplying and Dividing Fractions

Work out the following. Answers should be simplified and written as mixed numbers where necessary

(a) $\frac{1}{2}\times\frac{1}{5}$ (b) $\frac{1}{2}\times\frac{3}{4}$ (c) $\frac{1}{4}\times\frac{3}{5}$

(d) $\frac{2}{3}\div\frac{5}{6}$ (e) $\frac{1}{10}\div\frac{4}{9}$ (f) $\frac{6}{11}\div\frac{5}{6}$

Work out the following. Answers should be simplified and written as mixed numbers where necessary

(a) $1\frac{2}{3}\times\frac{1}{4}$ (b) $4\frac{3}{5}\times1\frac{2}{3}$ (c) $3\frac{1}{8}\times2\frac{1}{2}$

(d) $\frac{2}{3}\div1\frac{4}{5}$ (e) $2\frac{1}{3}\div5\frac{1}{2}$ (f) $4\frac{1}{3}\div2\frac{9}{10}$

Percentages of Amounts

Calculate the following. You should **not** use a calculator to complete these questions.

(a) 10% of 70m (b) 25% of 16 seconds (c) 10% of 400kg (d) 50% of 26g

(e) 3% of $9000 (f) 40% of 75 seconds (g) 15% of 90 hours (h) 5% of 14kg

(i) 90% of 1250ml (j) 76% of £80,000 (k) 85% of 90 hours (l) 12% of £6

Calculate the following. You should use calculator methods to complete these questions

(a) 15% of 80ml (b) 9% of 205kg (c) 45% of £135 (d) 17% of 540km

(e) 0.3% of 44km (f) 85.2% of 6000 marks (g) 0.25% of $840 (h) 3.175% of 52g

Reverse Percentages

1. A camera costs £180 in a 10% sale. What was the pre-sale price?
2. After fuel prices rose by 15%, a family's annual heating bill was £1654. What would the bill have been without the price increase?
3. The cost of a holiday, including VAT at 20% is £540. What is the pre-VAT price?
4. The world's tiger population has decreased by 95% since 1910 and is now believed to be as low as 3200. If these figures are correct, what was the tiger population in 1910?
5. The sale price of a television is £420 after a 15% reduction. What was the price before the sale?
6. After a 6.5% pay rise an engineer's salary is £36,700. What was the salary before the increase?
7. Due to falling orders a company reduces its workforce by 12% to 792. What was the original number of employees?
8. An engine modification improved the fuel consumption of a car by 27% to 17.2 km per litre. What was the fuel consumption before the modification?

Compound Growth and Decay

1. If £500 is invested for 3 years at a rate of compound interest of 4% per annum, how much will be in the account after 3 years?
2. Dave invests £3000 at a rate of interest of 6% a year. How much is in his account after 5 years?
3. Annie invests £1500 at a rate of compound interest of 2.5% for 6 years. How much is in her account after the six years?
4. Harry invests £1000 at a rate of interest of 5% a year. After how many years will he have doubled his investment?
5. John buys a house for £219000. The house depreciates in value at 6% each year. What is the value of the house after 7 years?
6. Sam bought his car 13 years ago for £14000. It has depreciated at 26% each year. How much is it now worth?
7. The value of a car depreciates by 15% each year. At the end of 2007, the value of the car was £8490. Work out the value of the car at the end of 2010.
8. Bob's new machine for work cost him £6700. It will depreciate at 28% each year. After how many years will it be worth less than £1000?

Recurring Decimals to Fractions

Convert the following recurring decimals to fractions. You should give each answer in its simplest form

(a) $0.\dot{2}$ (b) $0.\dot{8}$ (c) $0.\dot{1}\dot{8}$

(d) $0.\dot{5}\dot{3}$ (e) $0.\dot{7}\dot{5}$ (f) $0.\dot{6}\dot{3}$

(g) $0.\dot{1}1\dot{2}$ (h) $0.\dot{3}3\dot{9}$ (i) $0.\dot{1}7\dot{1}$

Convert the following recurring decimals to fractions. You should give each answer in its simplest form. Think carefully about which parts are recurring.

(a) $0.2\dot{8}$ (b) $0.0\dot{3}$ (c) $0.9\dot{6}$ (d) $0.5\dot{2}\dot{1}$

(e) $0.3\dot{9}\dot{0}$ (f) $0.1\dot{2}3\dot{5}$ (g) $0.12\dot{6}$ (h) $0.50\dot{3}\dot{5}$

GCSE Higher Topic 8 Ratio Student Knowledge Organiser

Key words and definitions

Compound measure: Compound measures are measures that are made up of two or more other measures. For example, speed is a compound measure, It is made up of distance and time.

Ratio: A ratio shows how much of one thing there is compared to the other.

Direct proportion: Direct proportion is when two (or more) quantities increase or decrease in the same ratio.

Indirect proportion: Inverse proportion is when an increase in one quantity results in a decrease in another quantity.

Speed, density & pressure.

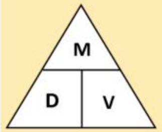
Speed Distance Time

Speed $= \frac{\text{Distance}}{\text{Time}}$

Distance = Speed x Time

Time $= \frac{\text{Distance}}{\text{Speed}}$

Mass Density Volume

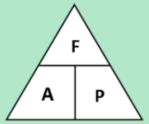


Volume $= \frac{\text{Mass}}{\text{Density}}$

Density $= \frac{\text{Mass}}{\text{Volume}}$

Mass = Density x Volume

Force Area Pressure



Pressure $= \frac{\text{Force}}{\text{Area}}$

Area $= \frac{\text{Force}}{\text{Pressure}}$

Force = Area x Pressure

Simplifying ratio.

Example 1

There are 15 fiction books and 10 non-fiction books on a shelf.
Write down the ratio of fiction books to non-fiction books in its simplest form.

1. Write down the ratio and divide both sides by the same number.
2. Stop when you can't divide any further.

$15 : 10$ ($\div 5$ both sides)
$3 : 2$

The simplest form is **3 : 2**

Dividing a ratio into parts.

Example 1

Nigel is going to split £40 between his two children. He shares the the money between Matthew and Emily in the ratio 2:3. How much money do Matthew and Emily receive?

40				
8	8	8	8	8

2 + 3 = 5 total shares

1 share = 40 ÷ 5 = **£8**

Matthew's share
2 shares = £8 × 2
= **£16**

Emily's share
3 shares = £8 × 3
= **£24**

Example 2

To make purple paint, red paint and blue paint are mixed in the ratio 3:5. Richard uses 720mL of paint altogether. How much blue paint does he use?

720							
90	90	90	90	90	90	90	90

3 + 5 = 8 total parts

1 part = 720 ÷ 8 = **90**

Red paint
3 shares = 90 × 3
= **270mL**

Blue paint
5 shares = 90 × 5
= **450mL**

Richard uses 450mL of blue paint.

Calculating a part of the ratio, given another.

Example 1

Laura makes some orange juice by mixing orange cordial and water in the ratio 3:10.
She uses 42mL of orange cordial.
How much water does she use?

14									
14	14	14	14	14	14	14	14	14	14

3 parts = **42mL**

1 part = 42 ÷ 3 = **14mL**

10 parts = 14 × 10 = **140mL**

Laura uses 140mL of water.

Example 2

Michael and Justine share some money in the ratio 5:3. Justine gets £108.
How much money did they share?

Michael	36	36	36	36	36
Justine	36	36	36		

3 parts = **£108**

1 part = 108 ÷ 3 = **£36**

5 + 3 = 8 total parts

8 parts = 36 × 8 = **£288**

Michael and Justine shared £288.

Direct proportion

Example 1

y is directly proportional to x.
Fill in the gaps in the table.

x	3	5	10	12	
y			25		100

1. Write the proportionality statement and make it into an equation. $y \propto x$, so $y = kx$
2. The table shows that when $x = 10$, $y = 25$. Use this to find k.
$25 = k \times 10$
$k = 25 \div 10 = 2.5$
So $y = 2.5x$
3. Use the equation to complete the table:

x	3	5	10	12	$100 \div 2.5 = \underline{40}$
y	$2.5 \times 3 = \underline{7.5}$	$2.5 \times 5 = \underline{12.5}$	25	$2.5 \times 12 = \underline{30}$	100

Example 2

m is directly proportional to e. Given that $m = 72$ when $e = 6$,

a) find the constant of proportionality,
1. Write the proportionality statement and make it into an equation. $m \propto e$, so $m = ke$
2. Use the given values to find k. $72 = k \times 6$, so $k = 72 \div 6$
$\underline{k = 12}$

b) calculate the value of e when $m = 37$.
1. Put the value of k from part a) into the equation $m = ke$. $m = 12e$
2. Substitute $m = 37$ into the equation and solve for e.
$37 = 12e$
$e = 37 \div 12 = \underline{3.08}$ (to 2 d.p.)

Inverse proportion

Example 1

y is inversely proportional to x.
Fill in the gaps in the table.

x	1	5	10	
y			20	100

1. Write the proportionality statement and make it into an equation. $y \propto \frac{1}{x}$, so $y = \frac{k}{x}$
2. The table shows that when $x = 10$, $y = 20$. Use this to find k.
$20 = \frac{k}{10}$
$k = 20 \times 10 = 200$
So $y = \frac{200}{x}$
3. Use the equation to complete the table:

x	1	5	10	$200 \div 100 = \underline{2}$
y	$200 \div 1 = \underline{200}$	$200 \div 5 = \underline{40}$	20	100

Example 2

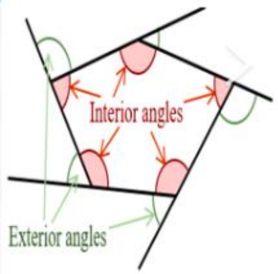
y is inversely proportional to x and $x = 4$ when $y = 15$.

a) Find y when $x = 10$.
1. Write the proportionality statement and make it into an equation. $y \propto \frac{1}{x}$, so $y = \frac{k}{x}$
2. Use the given values to find k. $15 = k \div 4$, so $k = 15 \times 4 = 60$
3. Put $k = 60$ into the equation. $y = \frac{60}{x}$
4. Substitute $x = 10$ into the equation and solve for y. $y = \frac{60}{x} = \frac{60}{10} = \underline{6}$

GCSE Higher Topic 9 Shapes and angles Student Knowledge Organiser

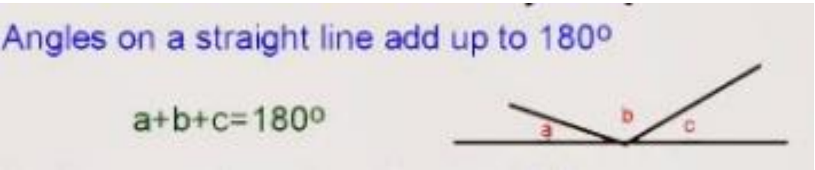
Key words and definitions

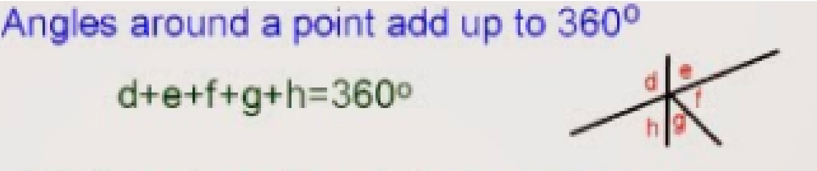
Interior angle – angle inside the shape
Exterior angle- angle on the outside.



Polygon- shape with all straight sides.
Regular polygon- all sides and angles are equal

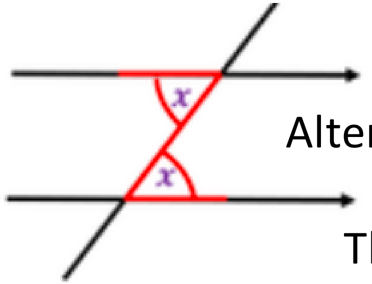
Angles around a point, on a straight line and in a triangle.





Angles in a triangle sum to 180°

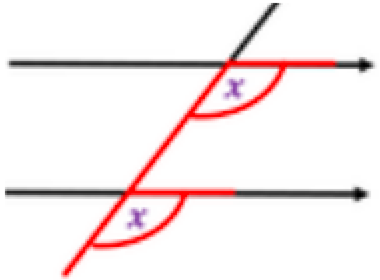
Alternate angles



Alternate angles are equal

They form a Z shape

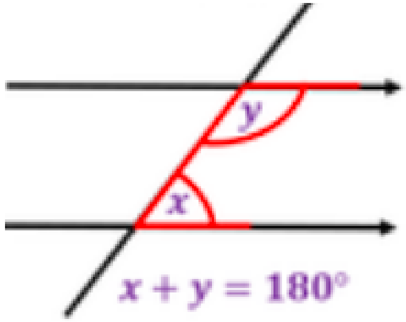
Corresponding angles



Corresponding angles are equal

They form a F shape

Co-interior angles



Co-interior angles add to 180°

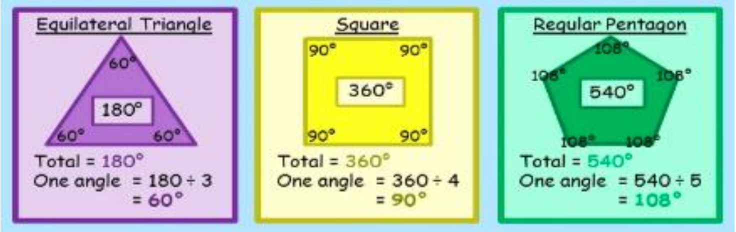
They form a C shape

Interior angles sum of polygons

A polygon with n number of sides

Angle sum = $(n-2)\times 180°$

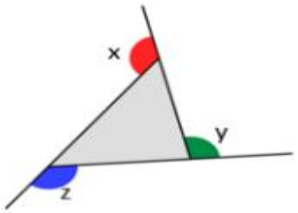
An angle of a regular polygon = $\dfrac{(n-2)\times 180°}{n}$



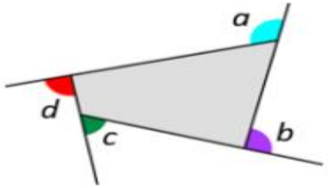
Exterior angles

The sum of the exterior angles of any polygon is 360°.

The exterior angle of a regular n-sided polygon is $\dfrac{360°}{n}$



x + y + z = 360°



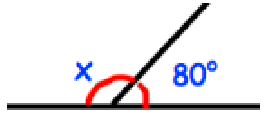
$a + b + c + d = 360°$

GCSE Higher Topic 9 Shapes and angles Student Knowledge Organiser

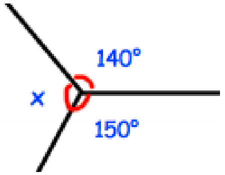
Angles around a point and on a straight line

Calculate x

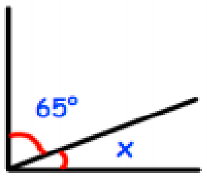
(a)



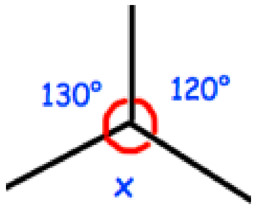
(b)



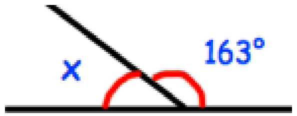
(c)



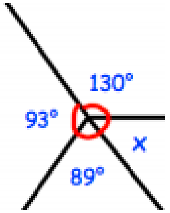
(d)



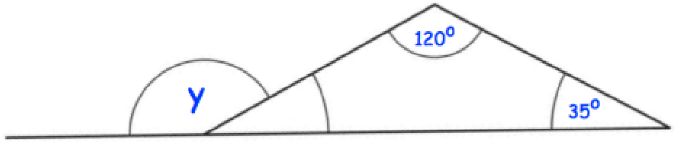
(e)



(f)



Calculate y

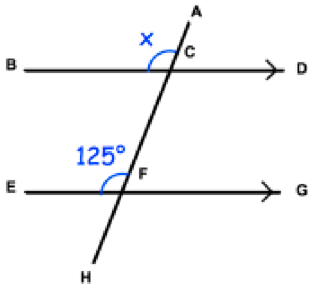


Work out the size of angle y.

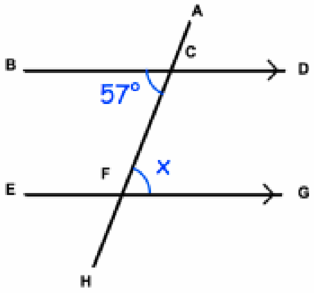
Angles in parallel lines

Calculate x

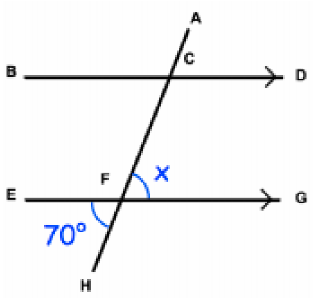
(a)



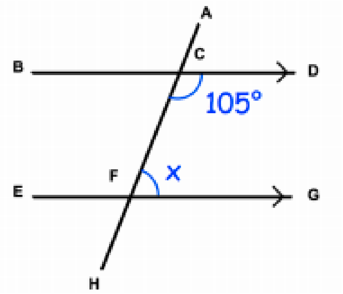
(b)



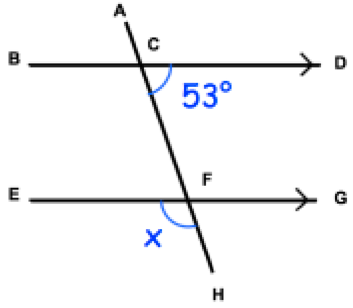
(c)



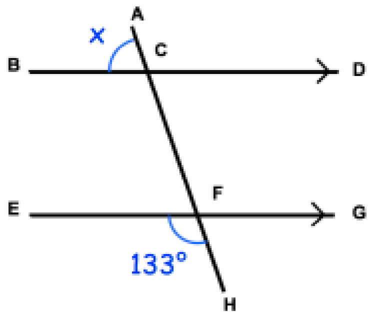
(d)



(e)



(f)



Interior angles

Work out the sum of the interior angles for polygons with

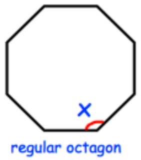
(a) 10 sides (b) 14 sides (c) 20 sides (d) 45 sides

Each of the polygons below are regular.
Calculate the size of each interior angle, x.

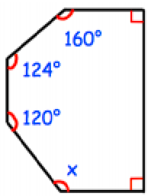
(a)

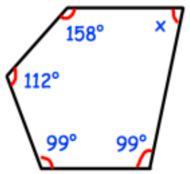
(b)

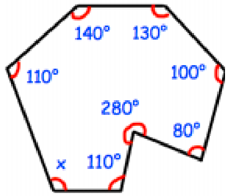
(c)



Find the missing angle in each irregular polygon



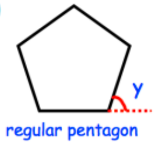




Exterior angles

Each of the polygons below are regular.
Calculate the size of each exterior angle, y.

(a)



(b)

(c)

Shown below is one interior angle from regular polygons.
Calculate how many sides the polygons have.

(a)

(b)

(c)

GCSE Higher Topic 10 Pythagoras Student Knowledge Organiser

Key words and definitions

Pythagoras: A Greek mathematician born in 570 BC

Right angled Triangle: A Triangle with one angle exactly 90 degrees.

Hypotenuse: The longest side of a right-angled triangle that has position always opposite the right angle

Isosceles Triangle: a triangle with two equal sides and two equal angles. There is a unique Isosceles triangle that is also right angled. Angles would be 45- 90- 45

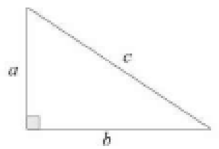
Pythagorean Triple : are three integers that form the sides of a right- angled triangle for example 3-4-5.

Pythagoras Theorem

Pythagoras Theorem links all three sides of a right angled Triangle together. Commonly we get two sides and need to find the third side

Pythagoras' Theorem

For any **right angled triangle**:

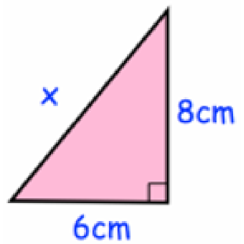


$$a^2 + b^2 = c^2$$

Used to find **missing lengths**.

a and b are the shorter sides, c is the **hypotenuse (longest side)**.

Finding the Longest Side (Hypotenuse)



Replace the values into the formula $a^2 + b^2 = c^2$

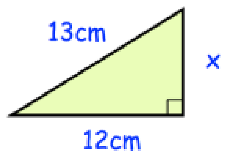
using $a = 6cm$ *and* $b = 8cm$ *to give* $6^2 + 8^2 = c^2$

$$36 + 64 = c^2$$

$$100 = c^2$$

This would give the missing side as 10cm

Finding a shorter side (not the hypotenuse)



Replace the values into the formula $a^2 + b^2 = c^2$

using $c = 13cm$ *and* $a = 12cm$ *to give* $12^2 + b^2 = 13^2$

$$144 + b^2 = 169$$

$$b^2 = 169 - 144$$

$$b^2 = 25$$

$$b = 5\ cm$$

This would give the missing side as 5 cm

Checking if a Triangle is Right Angled

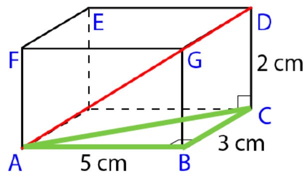
With an inaccurate diagram or just three lengths.

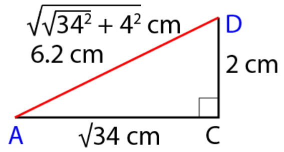
Carry out Pythagoras and see if the sum of the squares of the two shorter lengths are equal to the square of the longer side.

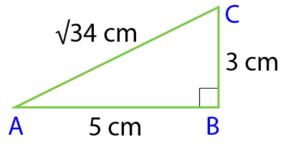
If Pythagoras Theorem holds true, these three sides form a right-angled triangle

Pythagoras In 3D

Commonly used as repeated Pythagoras. Using Pythagoras once to find the missing Length AC then again to find AD

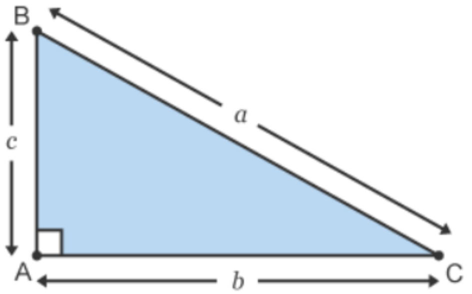






GCSE Higher Topic 10 Pythagoras and Surds Student Knowledge Organiser

Pythagoras

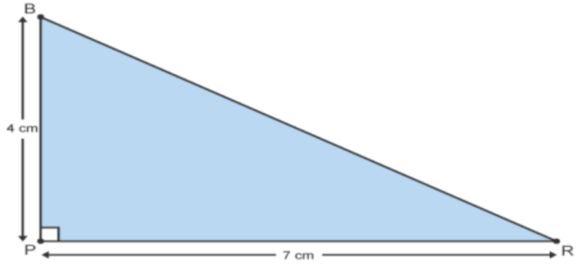


$$a^2 = b^2 + c^2$$

$$b^2 = a^2 - c^2$$

$$c^2 = a^2 - b^2$$

Work out the length of the line BR, correct to 1 decimal place.



A fireman has a ladder that is 13 metres long. If he wants to reach a window that is 12 metres above the ground, how far from the wall should he put the bottom of his ladder?

Peter's house is exactly 481m from school. To get home he walks 480m south and then he walks west. How far west does he have to walk?

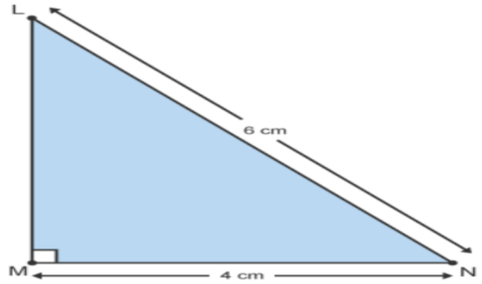
A triangle has sides of length 23.8cm, 31.2cm and 39.6cm.

Is this a right-angled triangle?

Show how you decide.

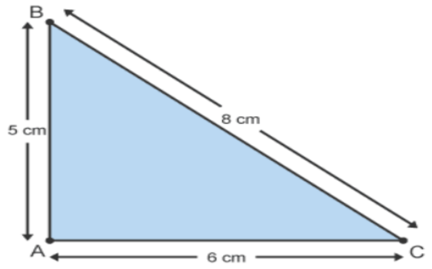
Pythagoras

Work out the length of the line LM, correct to 1 decimal place.



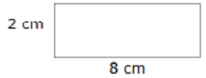
Which of the following triangles is right-angled?

a)

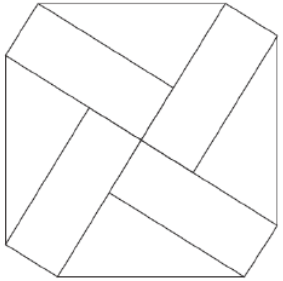


b)

Here is a rectangle.



The 8-sided shape below is made from 4 of these rectangles and 4 congruent right-angled triangles.



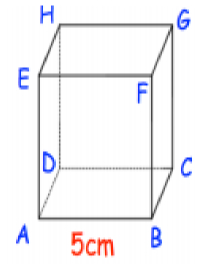
Work out the perimeter of the 8-sided shape.

3D Pythagoras

ABCDEFGH is a cube with side length 5cm.

(a) Work out the length of AC

(b) Work out the length of AG



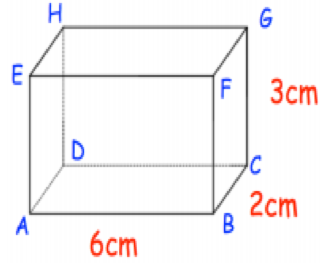
ABCDEFGH is a cuboid.
AB = 6cm, BC = 2cm and CG = 3cm.

(a) Work out the length of BG

(b) Work out the length of BD

(c) Work out the length of HC

(d) Work out the length of AG

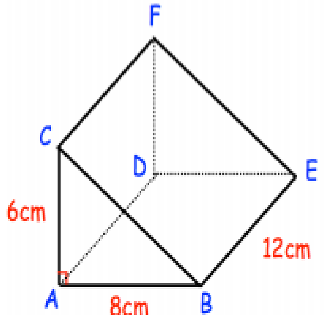


Shown is a triangular prism.
Triangle ABC is a right angle triangle.

(a) Work out the length of BC

(b) Work out the length of CD

(c) Work out the length of BF



GCSE Higher Topic 10 Surds Student Knowledge Organiser

Key words and definitions

Integer: a whole number (could be positive or negative)

Prime number: A prime number has only two factors - the number itself and 1. 1 is not a prime number

Rational Number: A number that can be whole or expressed as fraction $\frac{a}{b}$ where a and b are integers

Irrational Number : any number than cannot be expressed as fraction. Generally, means decimal values with no recurring/pattern

Square Number: the result of multiplying an integer by itself

Surd: An irrational number that is better expressed as a square root. If written as decimal they would continue forever with no pattern.

Surds are roots of numbers. Not every Root is a Surd

✓ $\sqrt{5}$ $5\sqrt{6}$ $\sqrt{2}$ $3\sqrt{2}$ $\sqrt{3}$ $\sqrt{11}$ $\sqrt{197}$

✗ $\sqrt{4}$ this can be simplified to 2, which is a rational number

$\sqrt[3]{27}$ this can be simplified to 3, which is a rational number

$(\sqrt{5})^2$ this can be simplified to 5, which is a rational number

Simplifying Surds – Method 2 is linked to Unit 1 work

Method 1

Simplify $\sqrt{24}$

Here we are looking for the largest square number which is also a factor of 24

Factors of 24:
1 x 24
2 x 12
3 x 8
4 x 6

So $\sqrt{24} = \sqrt{4 \times 6}$
$= \sqrt{4} \times \sqrt{6}$
$= 2\sqrt{6}$

Simplify $\sqrt{96}$

Here we are looking for the largest square number which is also a factor of 96

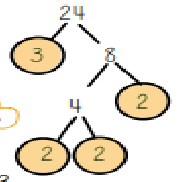
Factors of 96:
1 x 96
2 x 48
3 x 32
4 x 24
6 x 16
8 x 12

So $\sqrt{96} = \sqrt{6 \times 16}$
$= \sqrt{6} \times \sqrt{16}$
$= 4\sqrt{6}$

Method 2

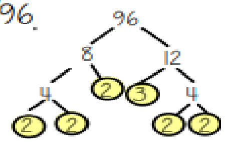
Simplify $\sqrt{24}$

Using prime factor decomposition and our knowledge that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, we can say



$24 = 2 \times 2 \times 2 \times 3$
So $\sqrt{24} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$
$= 2 \times \sqrt{2} \times \sqrt{3}$
$= 2\sqrt{6}$

Simplify $\sqrt{96}$



$96 = 2 \times 2 \times 2 \times 3 \times 2 \times 2$
So $\sqrt{96} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3} \times \sqrt{2} \times \sqrt{2}$
$= 2 \times 2 \times \sqrt{2} \times \sqrt{3}$
$= 4\sqrt{6}$

Adding and Subtracting Surds

$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ think of this like x + x, or 2 lots of x

coefficients are dealt with just like they are in algebra!

$4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$

$8\sqrt{2} - 5\sqrt{2} = 3\sqrt{2}$

$2\sqrt{3} - 7\sqrt{5}$ $\sqrt{3}$ and $\sqrt{5}$ are UNLIKE TERMS so this cannot be simplified any further

$4\sqrt{7} + 3\sqrt{10} - \sqrt{7} - 2\sqrt{10} = 3\sqrt{7} + \sqrt{10}$

$\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

$\sqrt{12} = \sqrt{4} \times \sqrt{3}$
$= 2 \times \sqrt{3}$
$= 2\sqrt{3}$

$\sqrt{27} = \sqrt{9} \times \sqrt{3}$
$= 3 \times \sqrt{3}$
$= 3\sqrt{3}$

It is important to try and simplify your surds before working with them so you don't miss things like this!

Multiplying and Dividing Surds

$\sqrt{2} \times \sqrt{5} = \sqrt{2 \times 5} = \sqrt{10}$

$\sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$

$\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$

$\boxed{\sqrt{a} \times \sqrt{a} = a}$

$\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = \sqrt{4} = 2$

$\sqrt{5} \times \sqrt{5} = \sqrt{5 \times 5} = \sqrt{25} = 5$

$\sqrt{10} \div \sqrt{2} = \sqrt{10 \div 2} = \sqrt{5}$

$\boxed{\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}}$

$\sqrt{12} \div \sqrt{3} = \sqrt{12 \div 3} = \sqrt{4} = 2$

Expanding Single and Double Brackets

Example 1

Expand and simplify $\sqrt{3}(2 + \sqrt{6})$

X	2	$+\sqrt{5}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{18}$

$= 2\sqrt{3} + \sqrt{18}$
$= 2\sqrt{3} + 3\sqrt{2}$

$\sqrt{18} = \sqrt{9} \times \sqrt{2}$
$= 3\sqrt{2}$

Always remember to check if you can simplify your surds

Example 2

Expand and simplify $\sqrt{3}(3\sqrt{8} - 2\sqrt{2})$

X	$3\sqrt{8}$	$-2\sqrt{2}$
$\sqrt{3}$	$3\sqrt{24}$	$-2\sqrt{6}$

$24 = 2 \times 2 \times 2 \times 3$
So $\sqrt{24} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$
$= 2 \times \sqrt{2} \times \sqrt{3}$
$= 2\sqrt{6}$

$= 3\sqrt{24} - 2\sqrt{6}$
$= 6\sqrt{6} - 2\sqrt{6}$
$= 4\sqrt{6}$

Example 3

Expand and simplify $(1 + \sqrt{3})(\sqrt{2} - 1)$

We can treat this just like we do double brackets in algebra

X	1	$+\sqrt{3}$
$\sqrt{2}$	$\sqrt{2}$	$\sqrt{6}$
-1	-1	$-\sqrt{3}$

none of these are 'like terms' so we cannot simplify anymore!

$= \sqrt{2} - \sqrt{3} + \sqrt{6} - 1$

GCSE Higher Topic 10 Surds Student Knowledge Organiser

Surds are almost exclusively a non- Calculator Topic at GCSE. You can use a calculator to check your answers. Type your question into the calculator, type your answer in, compare them.

Simplifying Surds – Exam Questions

1) $\sqrt{12}$
2) $\sqrt{50}$
3) $\sqrt{72}$
4) $\sqrt{60}$
5) $\sqrt{28}$
6) $\sqrt{96}$
7) $\sqrt{108}$
8) $\sqrt{32}$

Adding and Subtracting Surds

1) $2\sqrt{3} + 3\sqrt{3}$
2) $7\sqrt{7} - 3\sqrt{7}$
3) $7\sqrt{5} - 3\sqrt{5}$
4) $2\sqrt{7} - 3\sqrt{7}$
5) $2\sqrt{32} + 3\sqrt{2}$
6) $2\sqrt{27} - 3\sqrt{3}$
7) $2\sqrt{125} - 3\sqrt{80}$
8) $3\sqrt{24} - 3\sqrt{6}$
9) $\sqrt{108} + 2\sqrt{300}$
10) $5\sqrt{7} + 3\sqrt{28}$
11) $5\sqrt{294} - 3\sqrt{216}$

Multiplying and Dividing Surds

1) $\sqrt{12} \times \sqrt{6}$
2) $\sqrt{50} \times \sqrt{8}$
3) $\sqrt{14} \times \sqrt{28}$
4) $\sqrt{30} \times \sqrt{10}$
5) $\sqrt{15} \times \sqrt{45}$
6) $\sqrt{18} \times \sqrt{15}$
7) $\sqrt{120} \times \sqrt{15}$
8) $\sqrt{32} \times \sqrt{8}$
9) $3\sqrt{2} \times \sqrt{2}$
10) $5\sqrt{5} \times \sqrt{5}$
11) $2\sqrt{3} \times 3\sqrt{3}$

Expanding Single and Double Brackets

1) $\sqrt{2}(1 + \sqrt{2})$
2) $\sqrt{3}(2 - \sqrt{3})$
3) $\sqrt{3}(2\sqrt{3} + 1)$
4) $\sqrt{2}(3\sqrt{2} - 2)$
5) $2\sqrt{2}(1 + 2\sqrt{2})$
6) $3\sqrt{2}(2 - 2\sqrt{2})$
7) $2\sqrt{5}(3 + 4\sqrt{5})$
8) $6\sqrt{2}(\sqrt{2} - 6)$
9) $(1 + \sqrt{2})(2 + \sqrt{2})$
10) $(2 - \sqrt{3})(2 + \sqrt{3})$
11) $(\sqrt{3} + 2)(2\sqrt{3} + 1)$

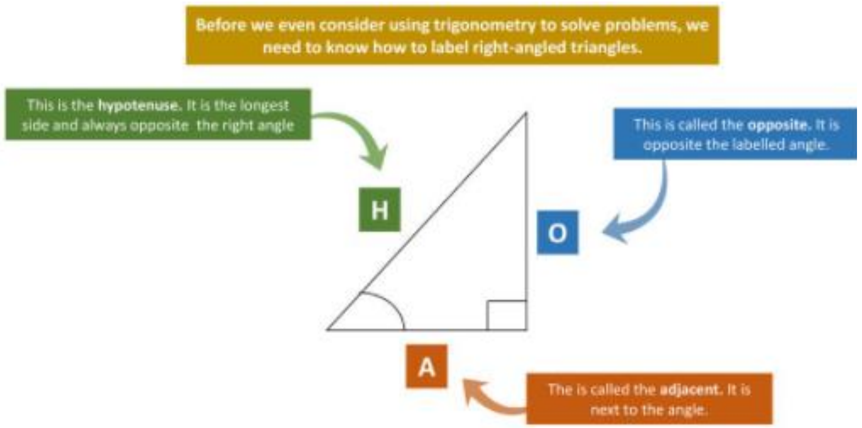
GCSE Higher Topic 11 Trigonometry Student Knowledge Organiser

Key words and definitions

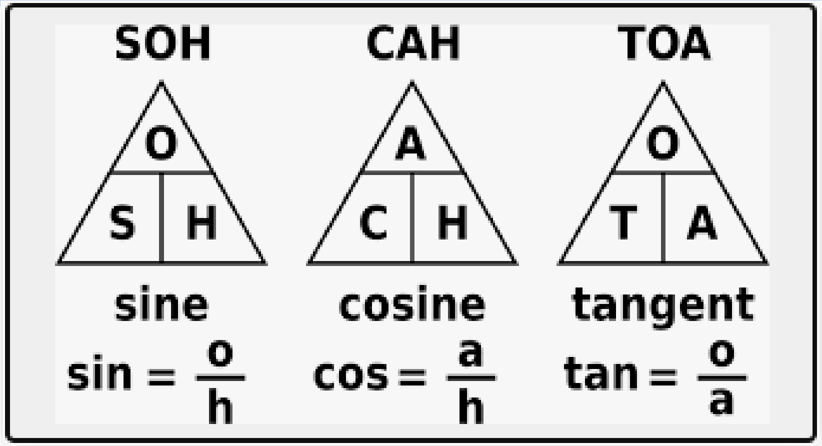
The HYPOTENUSE (h) is the longest side opposite the right angle
The OPPOSITE side(o) is opposite the angle in the question.
The ADJACENT side (a) is next to the angle in the question
Trigonometric ratios, SIN = $\frac{O}{H}$, COS = $\frac{A}{H}$, TAN = $\frac{O}{A}$
INVERSE trig ratios, used to find the angles in a triangle,

$\sin^{-1} = \frac{O}{H}$ $\quad \cos^{-1} \frac{A}{H}$ $\quad \tan^{-1} \frac{0}{A}$

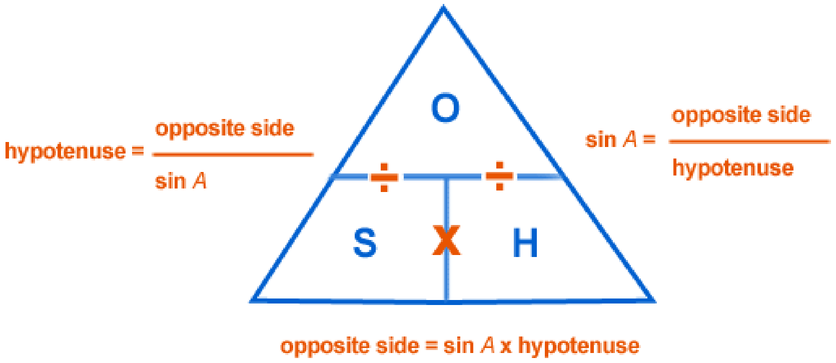
Labelling a right-angled triangle



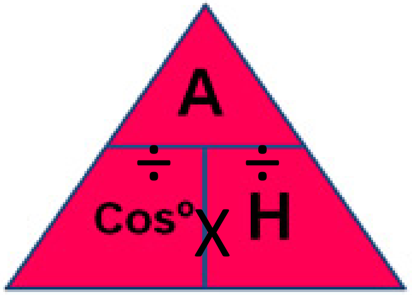
Trigonometric ratios



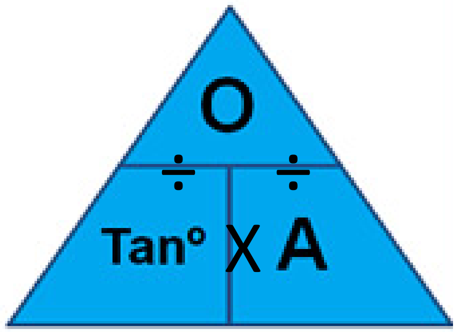
SINE (sin)



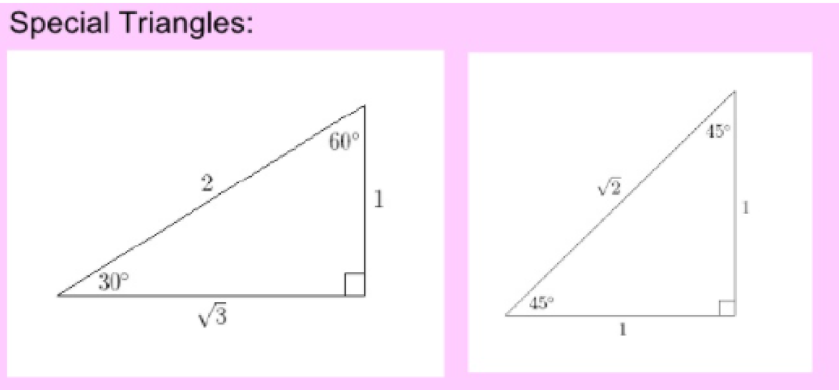
COSINE (cos)



TANGENT (tan)



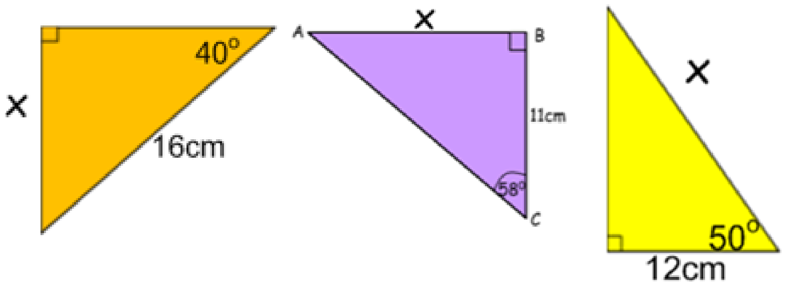
Special triangles, exact values



GCSE Higher Topic 11 Trigonometry Student Knowledge Organiser

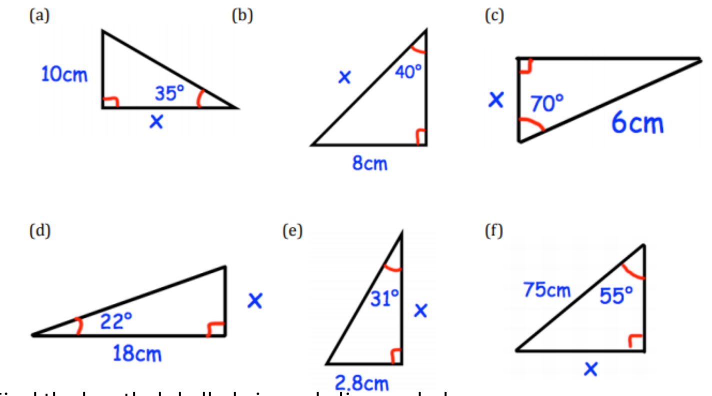
Missing Sides – Which ratio?

Label the sides of the following triangles. Which ratio would you use to calculate the missing side lengths in each one?

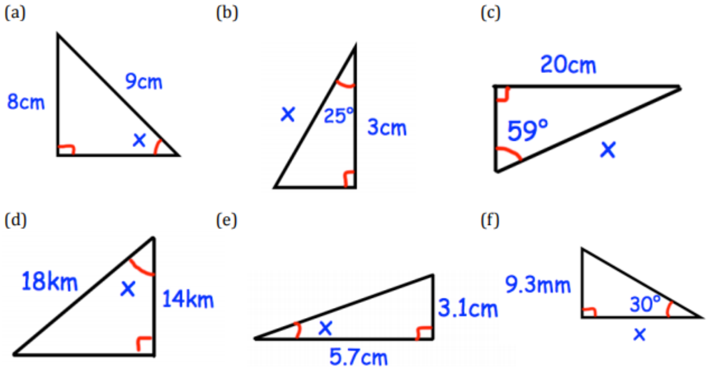


Finding Missing Side Lengths

Find the lengths labelled x in each diagram below:

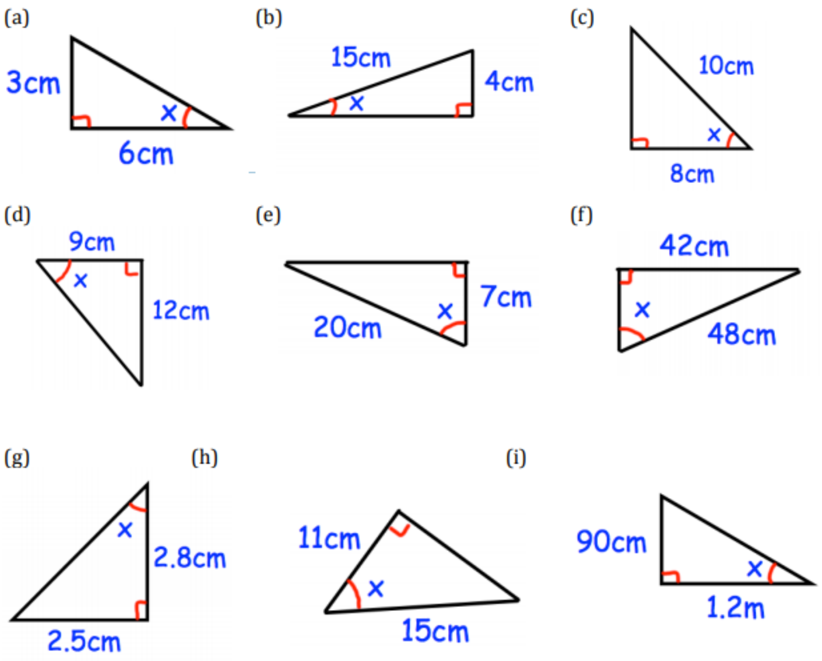


Find the lengths labelled x in each diagram below:



Finding angles

Calculate the size of the missing angles in the triangles below:



Exact values

Write down the exact values of the following:

a) sin 30°

b) cos 0 °

c) tan 45 °

d) sin 90 °

e) sin 0 °

f) cos 60 °

g) tan 0 °

h) sin 45 °

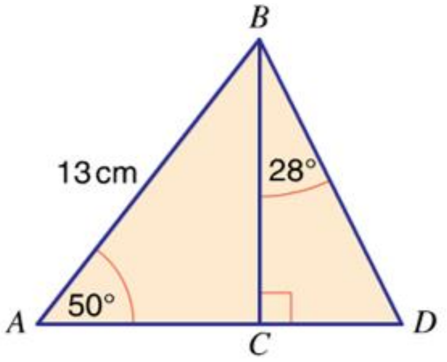
i) cos 30 °

j) tan 60 °

k) cos 90 °

l) sin 60 °

Multi step



Length of CD

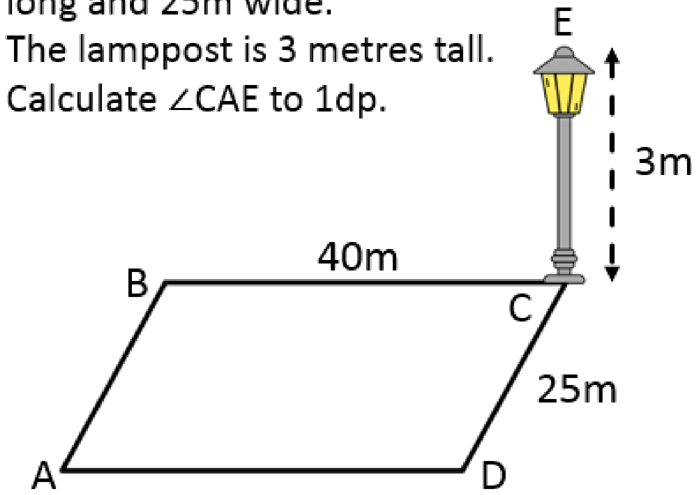
Length of AD

Pythagoras/ Trigonometry in 3d

A lamppost is located in the corner of a rectangular car park. The car park is 40m long and 25m wide.

The lamppost is 3 metres tall.

Calculate ∠CAE to 1dp.

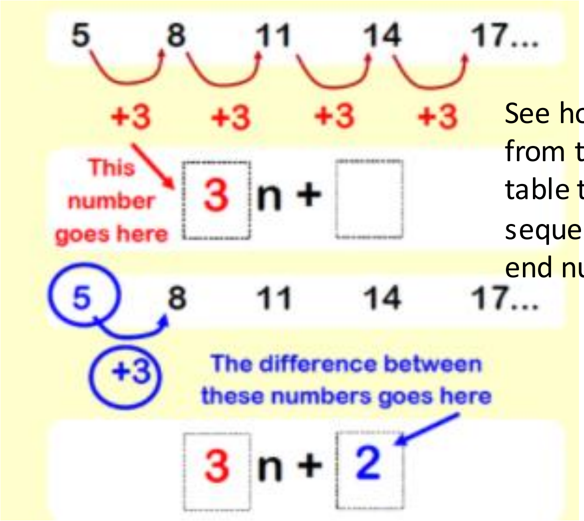


GCSE Higher Topic 12 Sequences Student Knowledge Organiser

Key words and definitions

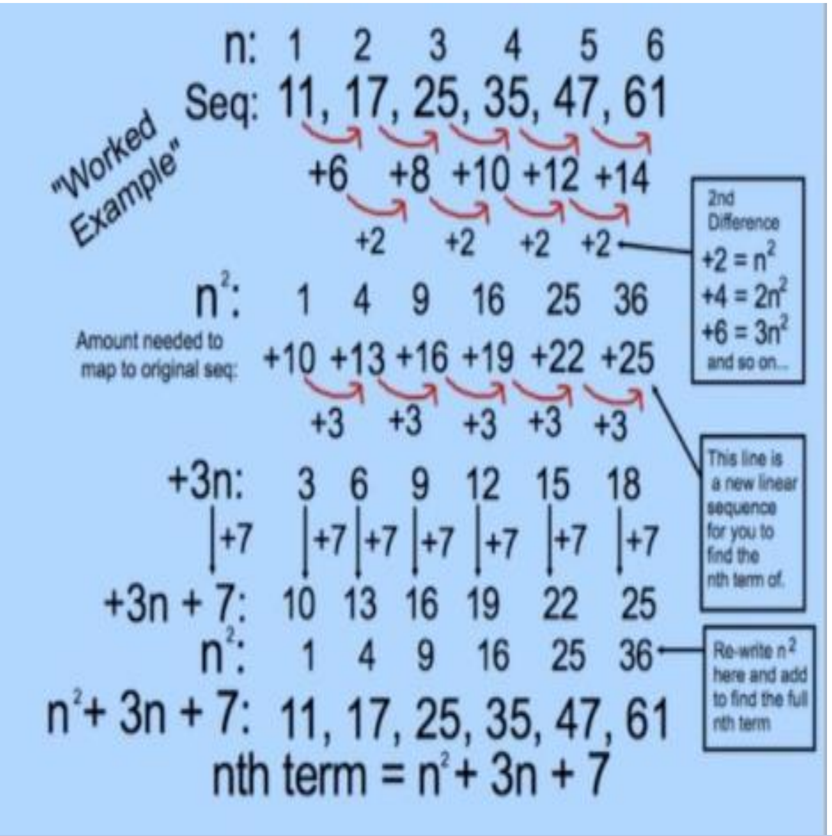
Sequence-terms or numbers put in a set order.
Term-the numbers/diagrams/letters in the sequence.
Arithmetic-a sequence where the difference between the terms in constant.
Geometric- a sequence where each term is found by multiplying the previous one by a fixed number.
Nth term- the rule of the sequence.

Finding the nth term of a linear sequence



See how you get from the 3 times table to the sequence to find the end number.

Finding the nth term of a quadratic sequence



Fibonacci sequences

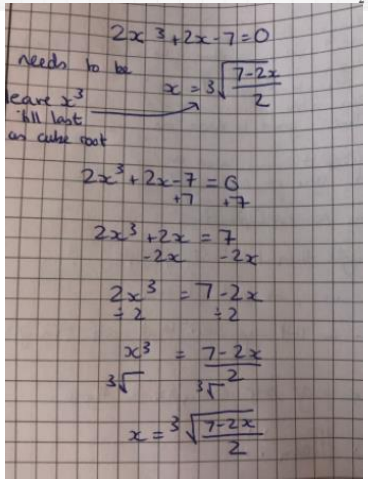
A Fibonacci sequence is found by adding the two previous terms:

Iteration

Iteration is when you put a starting value into a formula, complete the calculation and put that answer back in until you get the answer you need.

Substitute x_0 into the equation to find x_1.	$x_1 = \sqrt[3]{7-2(2)}$	$x_1 = 1.44224957...$
Put x_1 back into the equation to find x_2.	$x_2 = \sqrt[3]{7-2(1.44224957...)}$	$x_2 = 1.602535155...$
Repeat until two consecutive terms round to the same number to 1 d.p.	$x_3 = \sqrt[3]{7-2(1.602535155...)}$	$x_3 = 1.559796392...$

x_2 and x_3 both round to 1.6 to 1 d.p. so a solution is $\underline{x = 1.6}$.



You may also be asked to rearrange a formula in iteration.
This one said to show that the top equation could be wrote as the second.

GCSE Higher Topic 12 Sequences Student Knowledge Organiser

Finding the nth term of a linear sequence

Find a formula for the nth term for each of the following sequences.

7, 13, 19, 25...

5, 10, 15, 20...

–1, 1, 3, 5...

78, 69, 60, 51...

–10, –25, –40, –55...

Finding the nth term of a quadratic sequence

Find a formula for the nth term for each of the following sequences.

1, 5, 11, 19, 29...

7, 20, 39, 64, 95...

5, 12, 21, 32, 45...

6, 24, 52, 90, 138...

Iteration

Using $x_{n+1} = 8 - \frac{5}{x_n^2}$

with $x_0 = 1$

find the values of x_1 , x_2 , x_3 and x_4

Show that the equation $x^3 + 2x = 1$ can be rearranged to give

$$x = \frac{1}{2} - \frac{x^3}{2}$$

Using $x_{n+1} = \frac{20}{x_n^2} - 7$ with $x_0 = -9$

find the values of x_1 , x_2 and x_3

Fibonacci sequences

Find the next 3 terms in the Fibonacci style sequences:

$2, 4, 6, 10, \ldots$

$15, 23, 38, 62, \ldots$

$35, 60, 95, 155, \ldots$

$0.11, 2.32, 2.43, 4.75, \ldots$

$\frac{1}{11}, \frac{3}{11}, \frac{4}{11}, \frac{7}{11}, \ldots$

Finding the nth term of a cubic sequence

Use the Hegarty Maths clips to help find the nth term in the cubic sequences:

9, 16, 35, 72, 133...

3, 20, 63, 144, 275...

8, 21, 46, 89, 156...

GCSE Higher Topic 13 Probability Student Knowledge Organiser

Key words and definitions

Event: one or more outcomes from an experiment
Outcome: the result of an experiment. I
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets.
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence

Combinations

To find the total number of outcomes for two or more events, multiply the number of outcomes for each event together. This is called the product rule because it involves multiplying to find a product.

Example

A restaurant menu offers 4 starters, 7 main courses and 3 different desserts. How many different three-course meals can be selected from the menu?

Multiplying together the number of choices for each course gives $4 \times 7 \times 3 = 84$ different three-course meals.

Sample Space

A fair three sided spinner numbered from 1 to 3 is spun and a six sided die is rolled. The scores are added together. Put the results into the probability space diagram below.

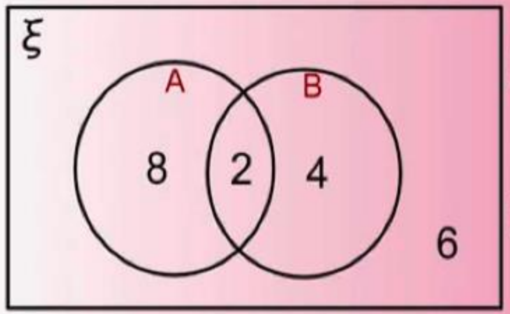
Die

Spinner	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9

What is the probability of the result being

1. 9 $\frac{1}{18}$
2. Less than 4 $\frac{3}{18} = \frac{1}{6}$
3. 7 $\frac{3}{18} = \frac{1}{6}$
4. 1 Zero

Venn Diagrams



Pr (Like AFL) $= \frac{10}{20}$

Pr (Like AFL OR Basketball) $= \frac{14}{20}$

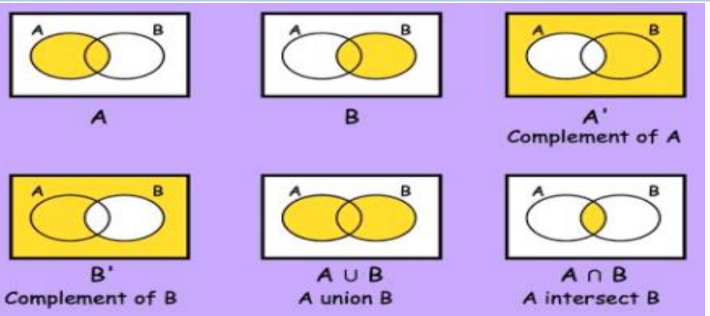
Pr (Like AFL AND Basketball) $= \frac{2}{20}$

Pr (Don't like AFL AND Basketball) $= \frac{18}{20}$

From a total of 20 students, 10 like AFL (A), 6 like Basketball (B) and 2 like both.

Represent this information in a Venn diagram.

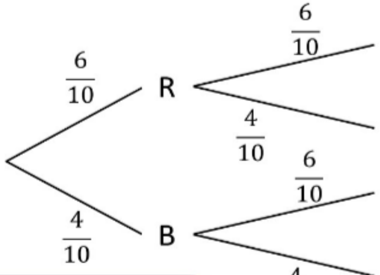
Set notation



Tree diagrams - unconditional

A bag contains 6 red cubes and 4 black cubes.
Jane randomly draws a cube. She **replaces it** and picks another.

What is the probability she picks two red cubes?



$RR \quad \frac{6}{10} \times \frac{6}{10} = \frac{36}{100}$

$RB \quad \frac{6}{10} \times \frac{4}{10} = \frac{24}{100}$

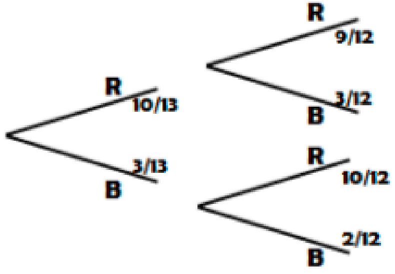
$BR \quad \frac{4}{10} \times \frac{6}{10} = \frac{24}{100}$

$BB \quad \frac{4}{10} \times \frac{4}{10} = \frac{16}{100}$

$\frac{36}{100}$

Tree Diagrams- conditional

1. A bag contains 10 red balls and 3 blue balls. A ball is chosen and not replaced before a second is chosen. Find the probability that,
a) two red balls are chosen
b) the balls are the same colour
c) the balls are different colours.



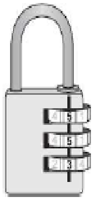
a) P(RR) = 10/13 x 9/12
= 90/156

b) P(RR) + P(BB)
= 10/13 x 9/12 + 3/13 x 2/12
= 90/156 + 6/156
= 96/156

c) P(RB) + P(BR)
= 10/13 x 3/12 + 3/13 x 10/12
= 30/156 + 30/156
= 60/156

GCSE Higher Topic 13 Probability Student Knowledge Organiser

Combinations



There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, 5
The three digit number 553 is one way the dials can be set, as shown in the diagram.

(a) Work out the number of different three digit numbers that can be set for the combination lock.

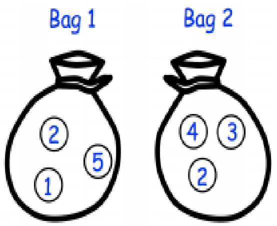
....

(2)

(b) How many of the possible three digit numbers have three different digits?

Sample Space

Question 1 Two bags, 1 and 2, each contain three counters.
In bag 1, the counters are labelled 1, 2 and 5.
In bag 2, the counters are labelled 2, 3 and 4.



A counter is drawn at random from bag 1 and a counter is drawn from bag 2.

The two numbers are multiplied together to give a score

(a) Complete the table to show all possible scores

(b) Find the probability of scoring a 6

(c) Find the probability of scoring a multiple of 4

(d) Find the probability of scoring an odd number

Bag 1 (columns), Bag 2 (rows)

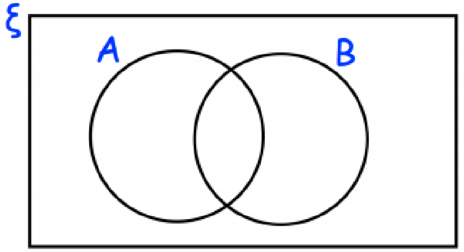
x	1	2	5
2			
3			
4			

Venn Diagrams and set notation

ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}

A = multiples of 3
B = multiples of 5

(a) Complete the Venn diagram

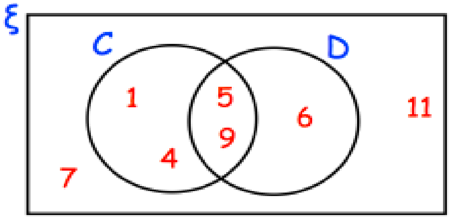


(3)

One of the numbers is selected at random.

(b) Write down P (A ∩ B)

2. Here is a Venn diagram



Write down the numbers that are in set

(a) D ......

(b) C ∪ D ......

(c) C' ......

Tree Diagrams

Megan and Rosie sit their driving tests.

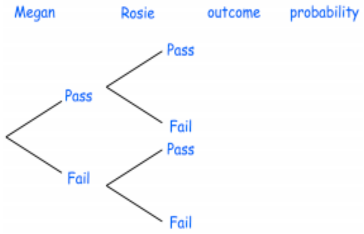
The probability that Megan passes the test is 0.8
The probability the Rosie fails the test is 0.3

(a) Copy and complete the tree diagram

(b) Find the probability that both women pass

(c) Find the probability that Megan fails and Rosie passes

(d) Find the probability that at least one woman passes

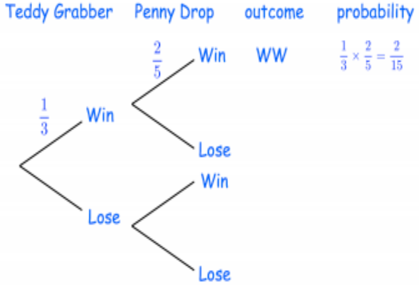


Harry goes to an arcade. He has one go on the Teddy Grabber and one go on the Penny Drop.
The probability that he wins on the Teddy Grabber is ⅓
The probability that he wins on the Penny Drop is ⅖

(a) Copy and complete the tree diagram

(b) Work out the probability that Harry loses on the Teddy Grabber and he also loses on the Penny Drop

(c) Work out the probability that Harry wins on exactly one machine



The probability that a bus arrives late is 0.1

Victor is travelling by bus on Monday and Tuesday.

(a) Show this information on a tree diagram

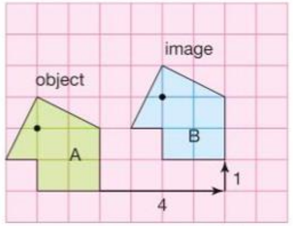
(b) Calculate the probability that the bus is on time both days.

GCSE Higher Topic 14 Transformations Student Knowledge Organiser

Key words and definitions

Enlarge – To make a shape larger (or Smaller)
Reflect – To produce an image of a shape as seen in a mirror
Rotate – To turn a shape about a centre point
Translate – To move a shape left or right and up or down
Column Vector - Used to describe a translation $\binom{x}{y}$ with x being left or right, y being up or down.
Congruence – Two shapes are congruent if they are both the same size and shape.
Similarity – Two shapes are similar if one is an enlargement of the other.
Scale Factor – By multiplying each side of a shape by this number you produce an image that has been enlarged.

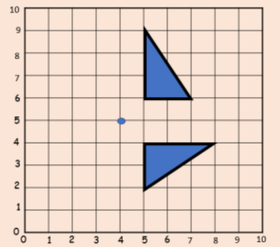
Translation



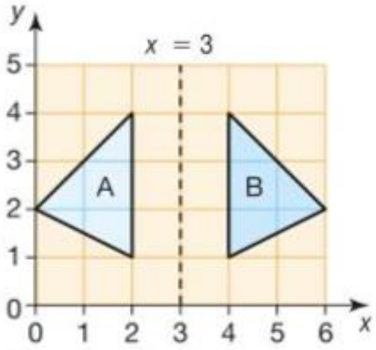
The shape is translated by $\binom{4}{1}$.

Rotation

Rotation 90° clockwise centre (4, 5)



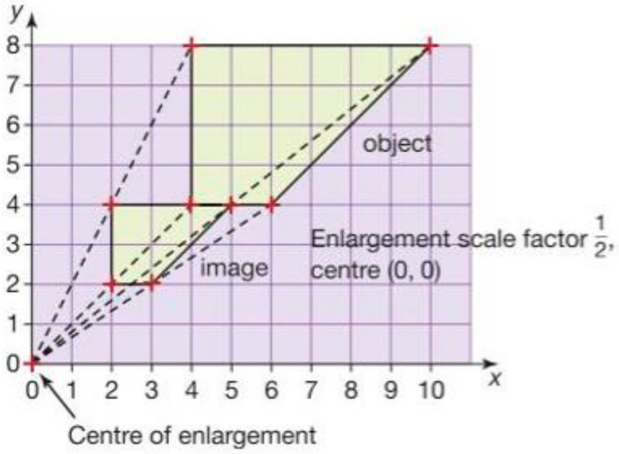
Reflection



The mirror line is $x = 3$.

Enlargement

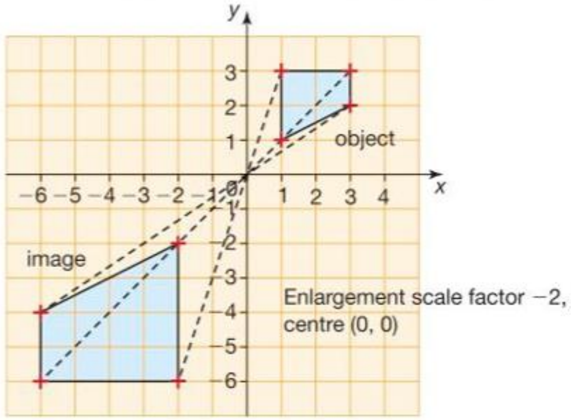
Enlargement with a fractional scale factor reduces the size of the shape.



Scale factor $\frac{1}{2}$: all lengths on the image are half the corresponding lengths on the object.

Enlargement – Negative Scale factor

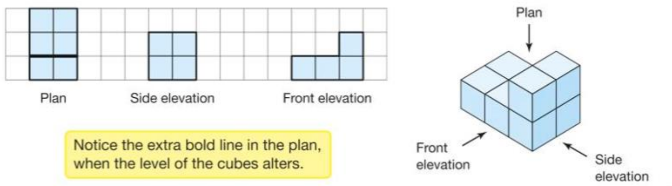
Enlargement with a negative scale factor produces a shape upside down on the opposite side of the centre.



Scale factor −2: all lengths on the image are twice the corresponding lengths on the object; the image is inverted.

Plans and Elevations

- A **plan** of a solid is the view from directly overhead (bird's eye view).
- An **elevation** is the view from the front or the side of the solid.



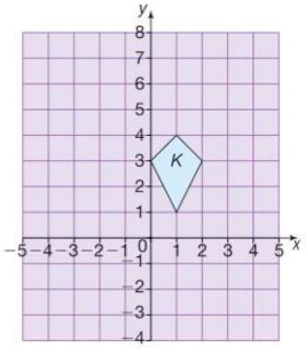
GCSE Higher Topic 14 Transformations Student Knowledge Organiser

Translation

Copy this diagram.

a Translate the kite K by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$. Label the image L.

b Translate the kite K by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$. Label the image M.

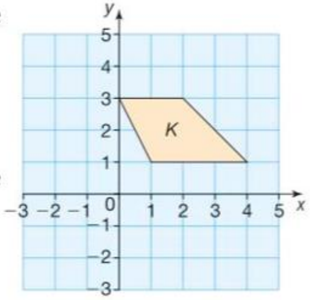


Rotation

Copy this diagram.

a Rotate the kite K through 180° about (1, 1). Label the image L.

b Rotate the kite K through −90° about (2, 0). Label the image M.

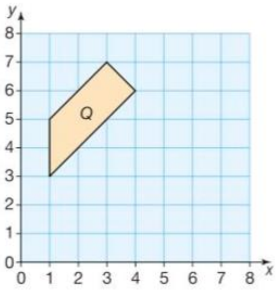


Reflection

Copy this diagram and extend the y-axis to −8.

a Reflect the quadrilateral Q in the x-axis. Label the image R.

b Reflect the quadrilateral Q in the line $y = x$. Label the image S.



Describing Transformations

Describe fully the transformation that maps

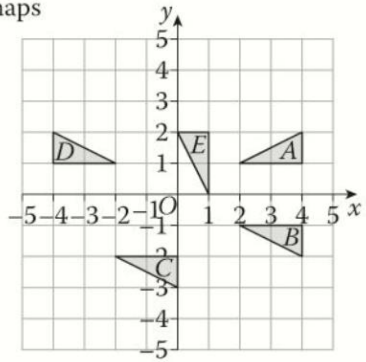
a triangle A to triangle B

b triangle C to triangle B

c triangle B to triangle D

d triangle D to triangle A

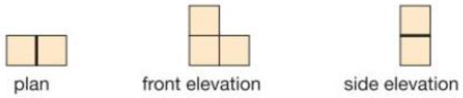
e triangle E to triangle A.



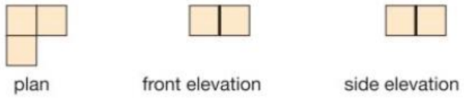
Plans and Elevations

The plan, front elevation and side elevation are given for these solids made from cubes. Draw a 3D sketch of each solid and state the number of cubes needed to make it.

a



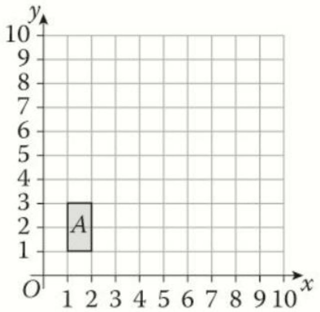
b



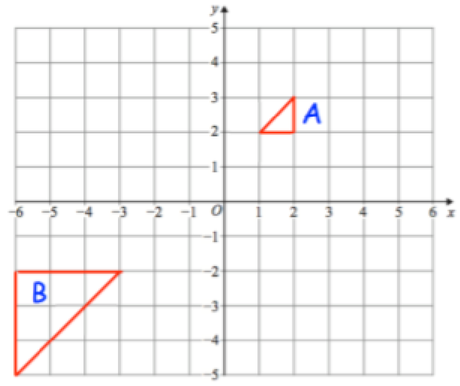
Enlargement

Copy this diagram.

Enlarge rectangle A by scale factor 3, centre (0, 0). Label the image B.



Describe fully the single transformation that takes shape A to shape B.



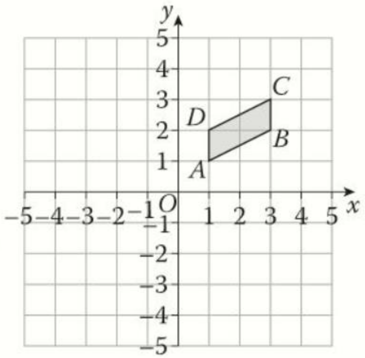
Combinations of Transformations

Copy this diagram.

a Reflect $ABCD$ in the x-axis, and label the image $A'B'C'D'$.

b Rotate $A'B'C'D'$ by 180° about the origin, and label the image $A''B''C''D''$.

c Find the single transformation that maps $ABCD$ to $A''B''C''D''$.



GCSE Higher Topic 15 – Cumulative Frequency Student Knowledge Organiser

Key words and definitions

Population- Every person in a certain place (eg. school, town , country.)
Sampling- A method to select a smaller a group of people from a certain population, done to be more time efficient.
Bias- A method of sampling which is not fair, favouring one particular group of people.
Random- A selection process where the is no conscientious method applied, to try to ensure fairness.
Cumulative Frequency- The running total of all the frequencies, a C.F table is used to draw a Cumulative Frequency Graph.
Quartiles- The values (UQ/LQ) which are the middle of all values above/below the median.
Inter Quartile Range- Calculated by UQ-LQ. This value shows how spread out the dataset ignoring the outliers.

Sampling

We select a sample from a population to be more time a efficient. It is important though that the people selected in a sample are chosen fairly and represent the full population as accurately as possible.There are several different methods of Sampling, the common ones are:
Random: A sample which is chosen using a method which eliminates potential bias. E.g Drawing names from a hat, or using a random number/name generator on a calculator or computer.
Systematic: A sample where people a selected from a list at pre-planned regular intervals (eg. Selecting every 10th Person from the list).
Stratified: A method which ensures that the sample represents the same proportions as the initial population. Eg. If 15% of the initial population are women aged between 30 and 40 years, then 15% of the sample need to be women aged between 30 and 40 years.
Once the grouping has been selected on a stratified sample then the people can be selected randomly or systematically,

Stratified sample model answer

2. There are 180 employees in a school.
The table shows the number of each type of employee in the school.

Teachers	Teaching Assistants	Admin	Other
94	16	41	29

(a) A stratified sample of size 50 is required.
Calculate the number of each type of employee that should be chosen.

$\frac{94}{180} \times 50 = 26.1$

$\frac{16}{180} \times 50 = 4.4\ldots$

$\frac{41}{180} \times 50 = 11.388\ldots$

$\frac{29}{180} \times 50 = 8.055\ldots$

Teachers ....26........
Teaching Assistants ....5........
Admin ....11........
Other ....8........
(3)

Cumulative frequency Table example

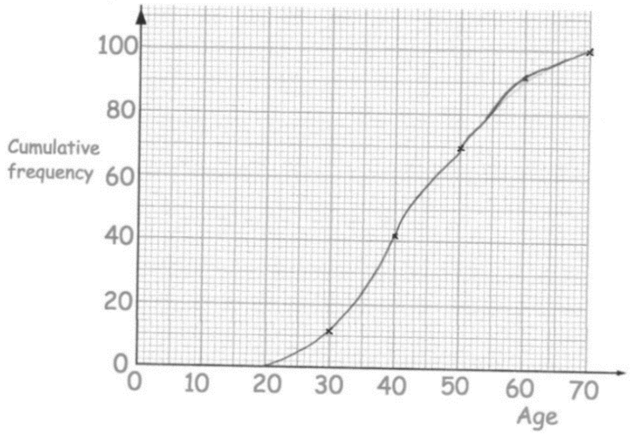
2. The ages of 100 teachers were recorded.
The table below shows this information.

Age, x years	Frequency	Cumulative frequency
20 < x ≤ 30	12	12
30 < x ≤ 40	30	42
40 < x ≤ 50	28	70
50 < x ≤ 60	22	92
60 < x ≤ 70	8	100

Cumulative frequency Graph: Key points

- **Plot C.F on the y-axis.**
- **Ensure you plot each point on your graph at the upper bounds of each category.**
- **Once you have plotted you graph you can draw on it to find Median Lower and Upper Quartiles. You can also calculate how many people scored under or over a certain mark.**

C.F Graph example.



(b) Draw a cumulative frequency graph for this information.
(2)

Boxplots (Use the C.F Graph to calculate Median, LQ and UQ)

GCSE Higher Topic 15 Cumulative Frequency Student Knowledge Organiser

Sampling

7 A tennis club wants to find out what facilities to offer.

The club's membership is

	18–30	31–50	over 50
Male	100	97	83
Female	140	133	147

Describe how to select a stratified sample by age and by gender.

8 There are 600 students in Years 6, 7 and 8 at a middle school. This incomplete table shows information about the students.

	Boys	Girls
Year 6	96	81
Year 7	87	102
Year 8		

A sample is chosen, stratified by both age and gender, of 60 of the 600 students.

a Calculate the number of Year 6 boys and Year 6 girls to be sampled.

In the sample there are nine Year 8 boys.

b Work out the least possible number of Year 8 boys in the middle school.

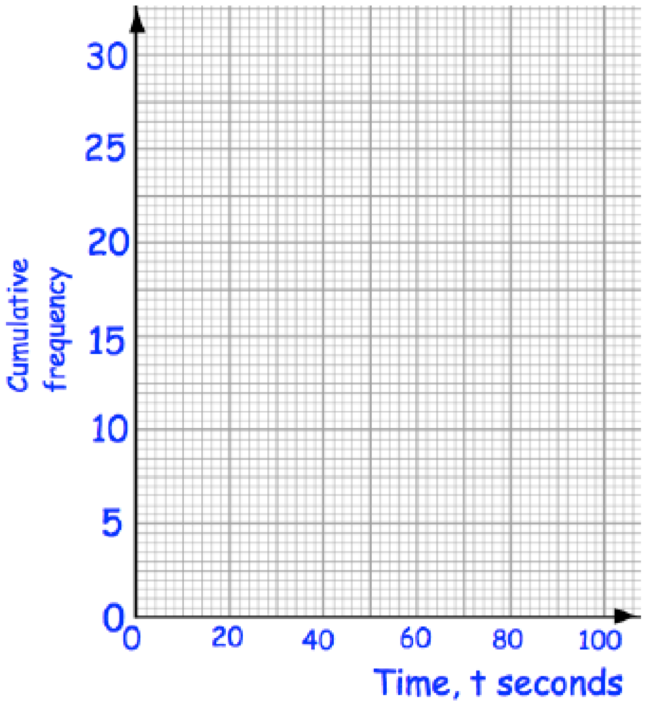
Cumulative Frequency .

10. A group of primary school students run an obstacle course.

The table below shows some information about their times.

Time, (t)	Cumulative frequency
$0 < t \leq 40$	4
$0 < t \leq 60$	11
$0 < t \leq 70$	16
$0 < t \leq 80$	22
$0 < t \leq 100$	30

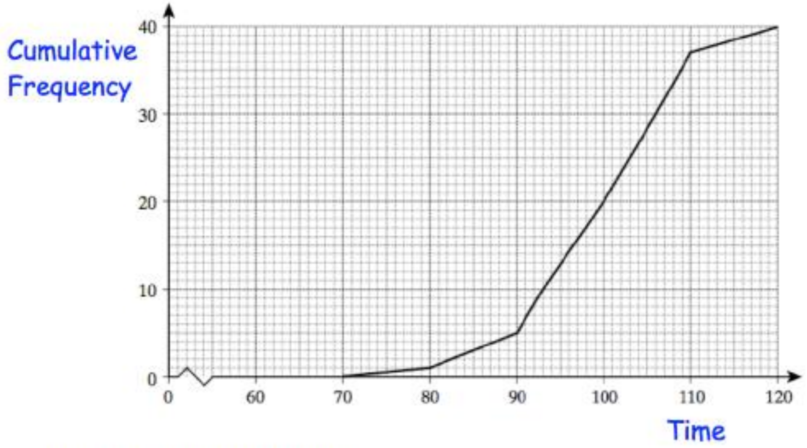
(a) On the grid, draw a cumulative frequency graph for this information.



(2)

Cumulative frequency and Boxplots.

8. 40 students complete a puzzle.
The time taken, in seconds, is recorded.
The cumulative frequency diagram shows the information about the times taken.



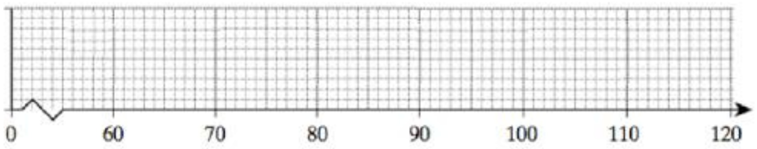
(a) Find the median time taken.

........................seconds
(1)

(b) Find the inter-quartile range.

........................seconds
(2)

(c) Complete a box plot for times taken.



(2)

GCSE Mathematics Higher - Topic 16 Equations 2

Key words and definitions

Quadratic: an expression where the highest order term is x^2

Formula: an equation with more than 1 type of letter in it.

Brackets: mathematical punctuation that tells you which part of the equation to calculate first.

Product: the answer when you multiply things together.

Sum: the answer when you add things together.

Factorise: write an expression in terms of a common factor.

Expand: multiplying sets of brackets together.

Factorising

- Many quadratic equations can be solved by rearranging so that one side equals zero and then **factorising**.

EXAMPLE

Solve $x^2 = 2x + 15$

$x^2 - 2x - 15 = 0$ Make the equation equal to zero.

$(x + 3)(x - 5) = 0$ Find two numbers with sum -2 and product -15.

Either $x + 3 = 0$ or $x - 5 = 0$

$x = -3$ $x = 5$

EXAMPLE

Two numbers have a product of 105 and a difference of 8.

If the larger number is x, form and solve an equation to find the two numbers.

1. The two numbers are x and $x - 8$.
   So $x(x - 8) = 105$
2. $x^2 - 8x - 105 = 0$
3. $(x + 7)(x - 15) = 0$
   Either $x + 7 = 0$ or $x - 15 = 0$
   So $x = -7$ and $x - 8 = -15$ or $x = 15$ and $x - 8 = 7$.
4. Two answers for x lead to two answers for $x - 8$.
   The two numbers are -7 and -15 or 7 and 15.

Complete the square

EXAMPLE

Solve $y^2 - 10y + 20 = 0$ by completing the square.

$y^2 - 10y + 20 = 0$ Complete the square.

$(y - 5)^2 - 25 + 20 = 0$

$(y - 5)^2 - 5 = 0$

$(y - 5)^2 = 5$ Add 5 to both sides.

5 has positive and negative square roots.

$y - 5 = \sqrt{5}$ or $y - 5 = -\sqrt{5}$

Leave your answers in surd form.

$y = 5 + \sqrt{5}$ or $y = 5 - \sqrt{5}$

Expanding Brackets

RECAP

- To expand double brackets, you multiply each term in the second bracket by each term in the first bracket.

F L O

$(2x + 7)(3x - 4) = 6x^2 - 8x + 21x - 28$
$= 6x^2 + 13x - 28$

- To factorise into double brackets, look for two numbers that add to give the coefficient of x and multiply to give the constant.

EXPAND

$(x + 4)(x + 7)$ $x^2 + 11x + 28$

$4 + 7$ 4×7

FACTORISE

Quadratic Formula

- You can use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve a quadratic equation $ax^2 + bx + c = 0$.

EXAMPLE

Solve the quadratic equation $3x^2 - 5x = 1$.

Give your answers to 3 sf.

$3x^2 - 5x = 1$

$3x^2 - 5x - 1 = 0$

In the formula: $a = 3$, $b = -5$ and $c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times (-1)}}{2 \times 3}$

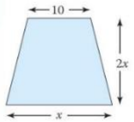
$x = \dfrac{5 \pm \sqrt{25 + 12}}{6}$

$x = \dfrac{5 + \sqrt{37}}{6} = 1.847127088\ldots = 1.85$ (to 3 sf)

Or $x = \dfrac{5 - \sqrt{37}}{6} = -0.180460421\ldots = -0.180$ (to 3 sf)

EXAMPLE

If the area of the trapezium is $400\,\text{cm}^2$, show that $x^2 + 10x = 400$ and find the value of x correct to 3 dp.



1. Area of trapezium $= \dfrac{(a + b)}{2}h$
   $A = \frac{1}{2}(x + 10) \times 2x = x(x + 10)$
   $400 = x(x + 10)$
   $x^2 + 10x = 400$ (as required)
2. $x^2 + 10x - 400 = 0$
3. The equation does not factorise, so use
   $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   $x = \dfrac{-10 \pm \sqrt{10^2 - 4 \times 1 \times -400}}{2}$
   $= 15.61552\ldots$ or $-25.61552\ldots$
4. Since x is a length, it must be positive, so $x = 15.616$ to 3 dp.

Solving Quadratic Equations

HOW TO

1. Use the information in the question to form a quadratic equation.
2. Rearrange the quadratic so that it equals zero.
3. Solve the quadratic by factorising, completing the square or using the quadratic formula.
4. Check that your answers make sense. You may need to reject one solution, depending on the context.

GCSE Mathematics Higher – Topic 16 Equations 2

Expanding Brackets

a $(x+2)(x+3)$ b $(p+5)(p+6)$

c $(w+1)(w+4)$ d $(c+5)^2$

e $(x+4)(x-2)$ f $(y-2)(y+7)$

g $(t+6)(t-2)$ h $(x-2)(x-5)$

i $(y-4)(y-10)$ j $(w-1)(w-2)$

k $(p-5)^2$ l $(q-12)^2$

Solve by factorising

a $x^2+7x+12=0$ b $x^2+5x-14=0$

c $x^2-4x-5=0$ d $x^2-5x+6=0$

e $2x^2+7x+3=0$ f $3x^2+7x+2=0$

g $2x^2+5x+2=0$ h $6y^2+7y+2=0$

i $x^2=8x-12$ j $2x^2+7x=15$

k $0=5x-6-x^2$ l $x(x+10)=-21$

Solve by Factorising (Difference of Two Squares)

a $x^2-16=0$ b $x^2-64=0$

c $y^2-25=0$ d $9x^2-4=0$

e $4y^2-1=0$ f $x^2=169$

g $4x^2=25$ h $36=9y^2$

Complete the square

(a) x^2+8x+1 (b) $x^2+10x+3$ (c) x^2+2x-1

(d) $x^2-6x-10$ (e) $x^2-4x-13$ (f) $x^2-12x+3$

(g) $x^2+14x+3$ (h) $x^2-2x-15$ (i) $x^2+4x-11$

(j) x^2+x-8 (k) x^2+3x+1 (l) x^2-7x-2

(m) x^2-9x-1 (n) $x^2+11x+3$ (o) $x^2-100x-25$

Solve by Completing the Square

a $x^2-12x+20=0$ b $x^2+2x-15=0$

c $x^2-4x-5=0$ d $x^2+2x+1=0$

e $x^2+2x-63=0$ f $x^2-14x+49=0$

g $x^2-8x=0$ h $y^2=1-12y$

i $p^2=3p+2$

Don't forget the two square roots.

Solve by using the Quadratic Formula

a $x(x+4)=9$

b $2x(x+1)-x(x+4)=11$

c $(3x)^2=8x+3$

d $y+2=\dfrac{14}{y}$

e $\dfrac{3}{x+1}+\dfrac{4}{2x-1}=2$

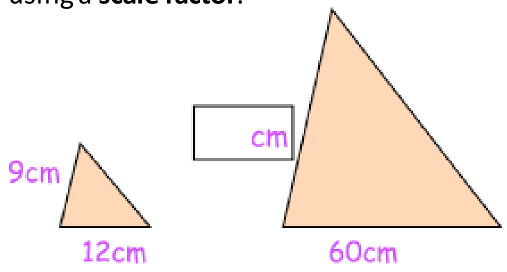
GCSE Higher Topic 17 Area and Volume 2

Key words and definitions

Volume: The amount of 3Dimensional space an object takes up.
Surface area – The sum of the areas of all the faces of a 3D object.
Similar : Two or more shapes are **similar** if they have the same shape, but are not necessarily the same size. The corresponding sides are in proportion and the corresponding angles are equal.
Scale factor – The size of an enlargement/reduction.
Sphere – A round 3Dimensional shape like a ball.
Pyramid – A 3D shape with triangular sides and a polygon base.
Cone – A 3D shape with a circular base joined to a point by a curved side.
Frustum – What is remaining of a cone or pyramid after its upper part has been cut off flat.
Convert – change the units of measurement.

Similar Lengths

Two shapes are similar if one is an enlargement of the other using a **scale factor**.



Scale factor $= 60 \div 12 = 5$

Missing length = corresponding length × scale factor

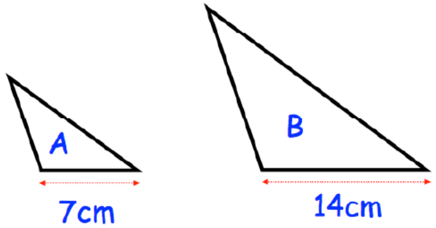
$= 9cm \times 5$

$= 45cm$

Similar Area

$$\textbf{Area scale factor} = (\textbf{scale factor})^2$$

Below are two similar triangles



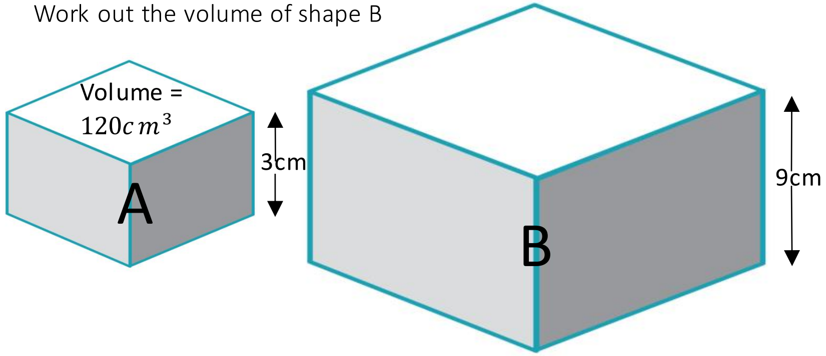
The area of triangle A is 20cm². Work out the area of triangle B.

Scale factor = $14 \div 7 = 2$
Area scale factor = $2^2 = 4$
Larger area = smaller area × area scale factor
$= 20cm^2 \times 4$
$= 80cm^2$

Similar Volume

$$\textbf{Volume scale factor} = (\textbf{scale factor})^3$$

Work out the volume of shape B



Scale factor = $9 \div 3 = 3$
Volume scale factor = $3^3 = 27$
$Larger\ volume = smaller\ volume \times volume\ scale\ factor$
$= 120cm^3 \times 27$
$= 3240cm^3$

Volume of non-prisms

$$\textbf{Volume of sphere} = \frac{4\pi r^3}{3}$$

Find the Volume of a Sphere with a radius of 3.2m

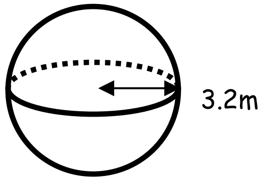
$Volume = \dfrac{4\pi r^3}{3}$

Sub in the value of r

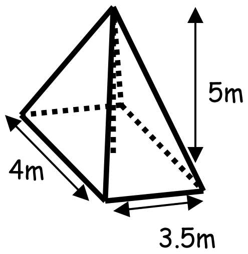
$Volume = \dfrac{4 \times \pi \times 3.2^3}{3}$

Calculate

$Volume = 137.26cm^3$



$$\textbf{Volume of cone/pyramid} = \frac{\textbf{area of base} \times \textbf{height}}{3}$$



$Volume = \dfrac{Area\ of\ Base \times Height}{3}$

Sub in values

$Volume = \dfrac{4 \times 3.5 \times 5}{3}$

Calculate

$Volume = 23.3cm^3$

GCSE Higher Topic 17 Area and Volume 2

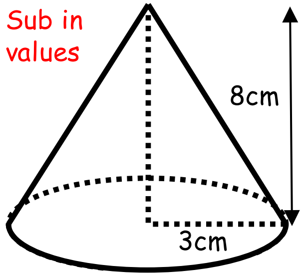
$$Volume = \frac{Area\ of\ Base \times Height}{3}$$

Sub in values

$$Volume = \frac{\pi \times 3^2 \times 8}{3}$$

Calculate

$$Volume = 75.4cm^3\ (24\pi)$$



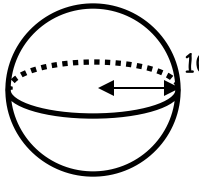
Surface area of non-prisms

$$\boxed{\textbf{Surface area of sphere} = 4\pi r^2}$$

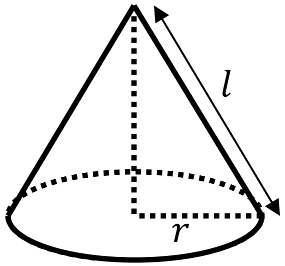
Calculate the surface area of a sphere with radius 10cm.

Surface area $= 4\pi \times 10^2$

$= 400\pi\,\text{cm}^2$



$$\boxed{\textbf{Surface area of cone} = \pi rl + \pi r^2 = \pi r(l + r)}$$

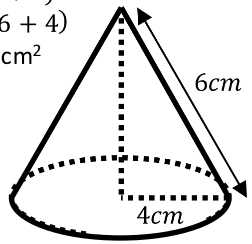


Calculate the surface area of a cone with radius 4cm and slant height 6cm.

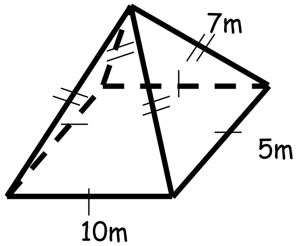
Surface area $= \pi r(l + r)$

$= 4\pi(6 + 4)$

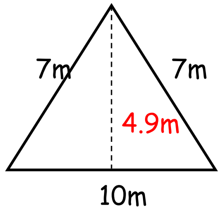
$= 40\pi\text{cm}^2$



Find the Surface Area of this Pyramid



A square based pyramid has 5 faces, 4 of which are identical triangles and a base which is a square. To find the surface area, we need to find the area of each of the faces and add them all together.



$c^2 - b^2 = a^2$ — Sub in c and b

$7^2 - 5^2 = a^2$ — Work out the left side

$24 = a^2$ — Square root

$4.9 = a$

base x height ÷ 2 — Sub in the base and height

(10 x 4.9) ÷ 2

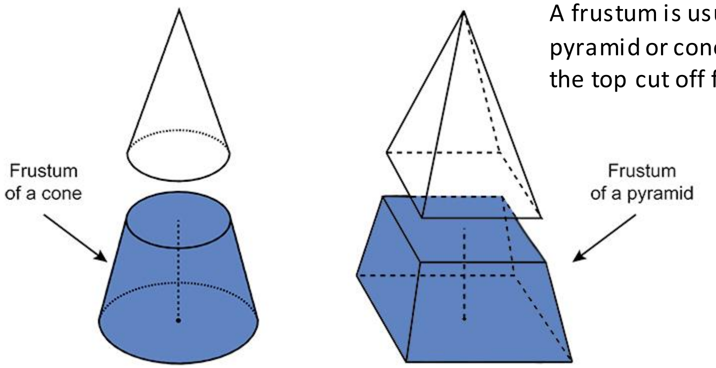
= 24.49m² — Multiply by 4 as there are 4 triangular sides

= 97.98m²

10 x 10 = 100m² ← Don't forget the base

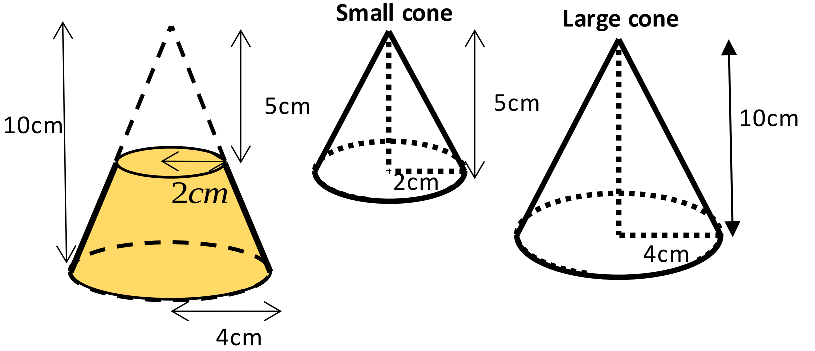
100m² + 97.98 = $\boxed{197.98\text{m}^2}$

Frustums



A frustum is usually a pyramid or cone with the top cut off flat

$$\boxed{Volume = Volume_{of\ large\ cone/pyramid} - Volume_{of\ small\ cone/pyramid}}$$



Volume of large cone	$\frac{1}{3} \times \pi \times 4^2 \times 10 = 167.6cm^3$
Volume of small cone	$\frac{1}{3} \times \pi \times 2^2 \times 5 = 20.9cm^3$
Volume =	$167.6 - 20.9 = 147cm^3$

We use this method for pyramid shaped frustums as well.

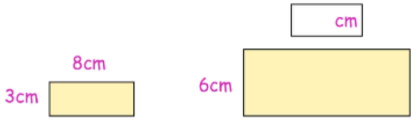
Sometimes we may have to use similarity or Pythagoras to find missing lengths or heights.

GCSE Higher Topic 17 Area and Volume 2

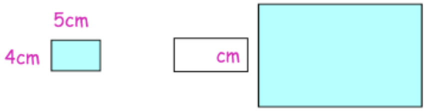
Similar Lengths

Question 1: Below are pairs of similar shapes. Find the missing lengths.

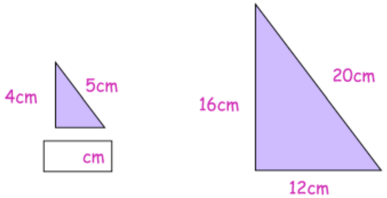
(a)



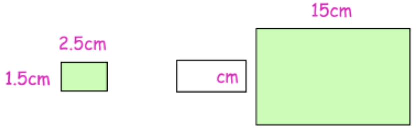
(b)



(c)



(d)



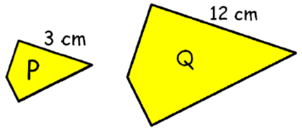
Similar Areas

Question 1:

Quadrilaterals P and Q are similar.

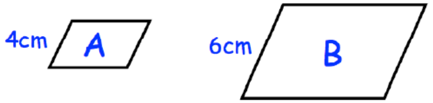
The area of quadrilateral P is 10cm².

Calculate the area of quadrilateral Q



Question 2:

Below are to similar parallelograms.



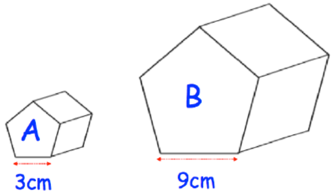
The area of parallelogram A is 28cm².

Work out the area of parallelogram B.

Similar Volumes

Question 1:

Below are two similar pentagonal prisms

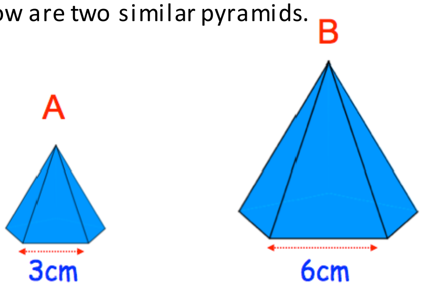


The volume of prism A is 15cm³.

Work out the volume of prism B

Question 2:

Below are two similar pyramids.



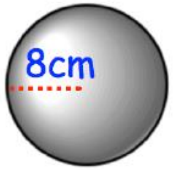
Pyramid A has a volume of 26cm³

Work out the volume of pyramid B.

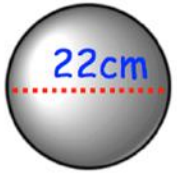
Volumes of non-prisms

Find the volume of the following objects, leave your answer to 1 decimal place

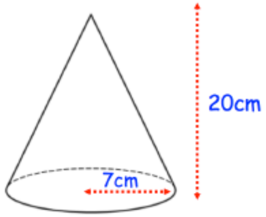
(a)



(b)



(a)



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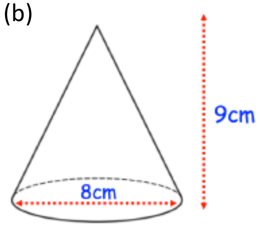
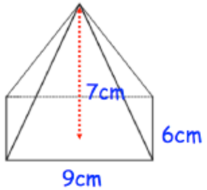


(a)



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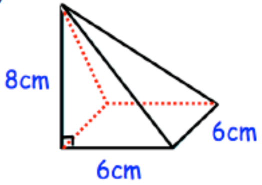
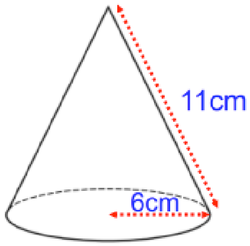


(a)



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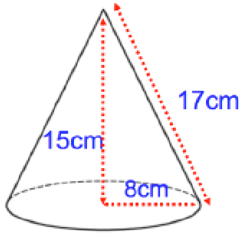
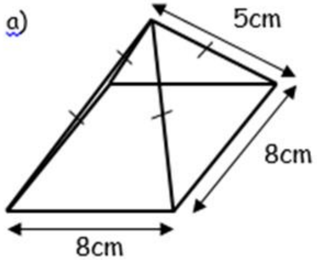
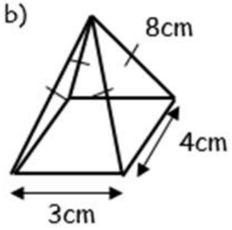


a)



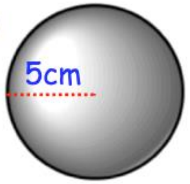
b)



Surface areas of non-prisms

Work out the surface areas of the following objects, leave your answer to 1 decimal place

(a)



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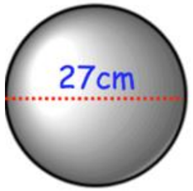
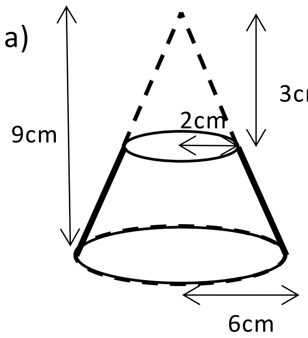


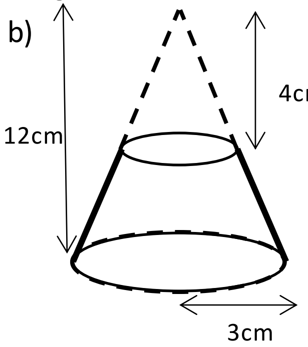
Frustums

Work out the volume of the following frustums

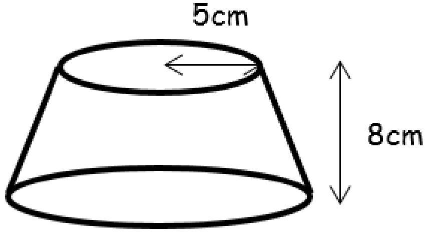
a)



b)



c)



GCSE Mathematics Higher - Topic 18 Graphs

Key words and definitions

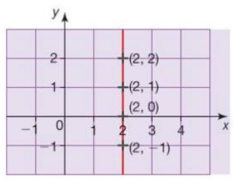
Parallel: Lines that have the same gradient and never meet.
Perpendicular: Lines that meet at a 90 degree angle.
Gradient: Steepness of a line.
y-intercept: Where a line crosses the y-axis.
Coordinate: How far along an axis a point is. In 2D space, a point will have two coordinates written as (x, y).
Plot: Draw points on a coordinate grid. When asked to plot a graph, you need to plot the points from the table of values and then join them together with a straight line.

Horizontal and Vertical Lines

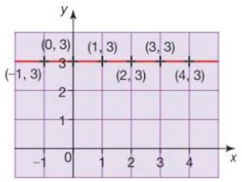
A straight line can be **diagonal**, **vertical** or **horizontal**.

The x-coordinate of every point on this vertical line is 2.

The y-coordinate can have any value.



The equation of the line is $x = 2$.



The equation of the line is $y = 3$.

The y-coordinate of every point on this horizontal line is 3.

The x-coordinate can have any value.

- Horizontal lines have equations of the form $y = c$.
- Vertical lines have equations of the form $x = c$.

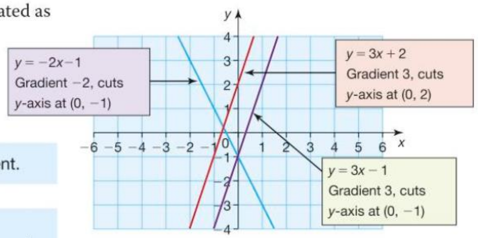
c stands for a number.

Equation of a Straight Line

- The equation of a straight line is of the form $y = mx + c$, where m is the **gradient** and c is the **y-intercept**.

The y-intercept is the y-value where the graph cuts the y-axis.

The **gradient** of a line segment is calculated as $\dfrac{\text{Change in the } y\text{-direction}}{\text{Change in the } x\text{-direction}}$.



- **Parallel** lines have the same gradient.
- If line A has gradient m, any line perpendicular to line A has gradient $-\frac{1}{m}$.

Plotting Straight Line Graphs

- The graphs of linear equations such as $y = 2x + 3$ are straight lines.

If you plot three points, you can tell if you have made a mistake.

One point must be wrong.

To plot a graph of a function:

- Draw up a table of values
- Calculate the value of y for each value of x
- Draw a suitable grid
- Plot the (x, y) pairs and join them with a straight line.

EXAMPLE

a Draw the graph of $y = 2x + 3$.

b Use the graph to find **i** the value of x when $y = 7$ **ii** the value of y when $x = 3\frac{1}{2}$.

Construct a table of values.

Choose four or five values, including negative values and zero.

a

x	−2	−1	0	1	2
y	−1	1	3	5	7

(−2, −1) (−1, 1) (0, 3) (1, 5) (2, 7)

Then plot the points and draw the line.

Make sure that your grid includes the smallest and largest y-values.

b **i** Find $y = 7$ on the y-axis. Draw a horizontal line to the graph line. Draw a vertical line to the x-axis. Read off the value of x.

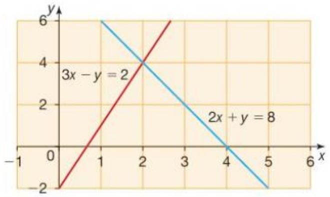
$x = 2$

ii Find $x = 3\frac{1}{2}$ on the x-axis. Draw a vertical line to the graph line. Draw a horizontal line to the y-axis. Read off the value of y.

$y = 10$

Solving Simultaneous Equations

You can solve **simultaneous** equations graphically.
A solution is at a point of **intersection**.
For example, for the equations $3x - y = 2$ and $2x + y = 8$, the lines intersect at (2, 4) so the solution is $x = 2$ and $y = 4$.



HOW TO

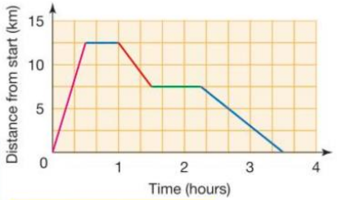
1. Use the information in the question to form a pair of simultaneous equations.
2. Solve the simultaneous equations using elimination, substitution or by drawing a graph.
3. Give your answers and check that they make sense.

Distance-Time Graphs and Velocity-Time Graphs

A distance–time graph shows information about a journey.

The gradient of a straight line in a distance–time graph is the speed of the object.

Velocity–time graphs also give information about a journey.



- The gradient of a straight line in a velocity–time graph is the **acceleration** of the object.
- The area under a line in a velocity–time graph is the distance travelled by the object.

Velocity is speed in a certain direction.

Equation of a Straight Line Example

EXAMPLE

Find the equation of the line perpendicular to $y = 2x - 1$ that passes through (4, 5).

$y = 2x - 1$ has gradient 2, so a line perpendicular to it has gradient $-\frac{1}{2}$.

$y = -\frac{1}{2}x + c$

At (4, 5): $5 = \left(-\frac{1}{2}\right) \times 4 + c$

$c = 7$

The equation is $y = -\frac{1}{2}x + 7$.

If the equation is not in the form $y = \ldots$, rearrange it first, for example

$3x + 2y = 12 \rightarrow 2y = -3x + 12 \rightarrow y = -\frac{3}{2}x + 6$

Now you can see that the gradient is $-\frac{3}{2}$ and the intercept is 6.

GCSE Mathematics Higher – Topic 18 Graphs

Solving Simultaneous Equations with Lines

Two lines intersect. One has gradient 4 and y-axis intercept 3. The other has gradient 6 and cuts the y-axis at (0, 1).

Find the point of intersection of the lines.

Drawing Straight Line Graphs from the Equation

Draw the graphs of these functions.

a $y = 3x - 2$ **b** $y = -2x + 4$

c $y = \frac{1}{2}x + 3$ **d** $y = 5 - x$

Drawing Vertical and Horizontal Lines

a $y = 4$ **b** $x = 5$ **c** $y = -2$

d $x = -2$ **e** $x = -4$

Finding the Equation of a Line

Find the equations of these lines.

a Gradient 6, passes through (0, 2)

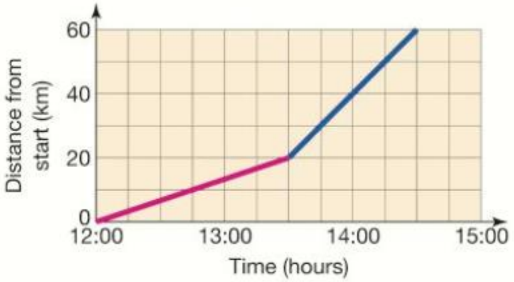
b Gradient -2, passes through (0, 5)

c Gradient -1, passes through $(0, \frac{1}{2})$

d Gradient -3, passes through (0, -4)

Distance-Time Graphs

The distance-time graph shows information about Lisa's coach journey.



a How far does she travel between

i 12:00 and 13:30

ii 13:30 and 14:30?

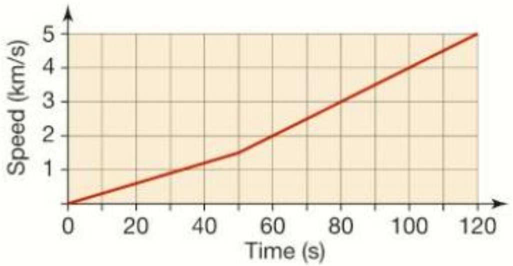
b How long does it take to travel

i 10 km from the start

ii 50 km from the start?

Speed-Time Graphs

A rocket accelerates in two stages as shown in the speed-time graph.



a Calculate the acceleration, in km/s², for

i stage 1 **ii** stage 2.

b Calculate the average acceleration for the whole journey.

GCSE Mathematics Higher - Topic 19 Constructions

Key words and definitions

Construct: Draw accurately with mathematical equipment.

Arc: A curved line, often drawn with a pair of compasses.

Perpendicular: Meeting at a 90 degree (right) angle.

Bisector: Dividing into two equal pieces.

Loci: Potential positions for an object on a diagram.

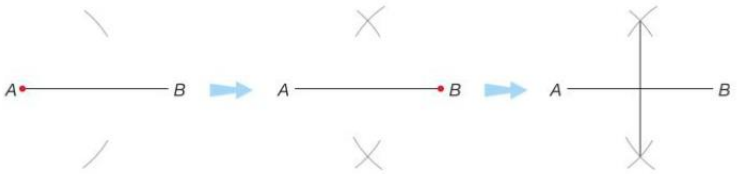
Region: A 2D space that satisfies certain criteria.

Equidistant: The same distance away.

Perpendicular Bisector

- The **perpendicular** bisector of a line bisects the line at right angles.

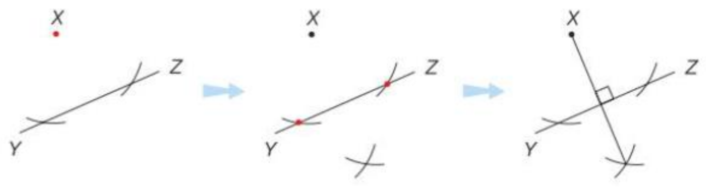
- To construct the perpendicular bisector of line AB



- All points on the perpendicular bisector of AB are equidistant from A and B.

Perpendicular From a Point

- To construct the perpendicular from a point X to a line YZ.



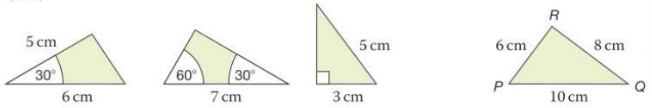
Start at the red dots.

Keep the same compass radius throughout the construction.

Constructing Triangles

You can **construct** a unique **triangle** when you know

two sides and the angle between them (SAS) or two angles and a side (ASA) or right angle, the hypotenuse and a side (RHS) or three sides (SSS).



Any two triangles constructed using any one of these four sets of information will be congruent.

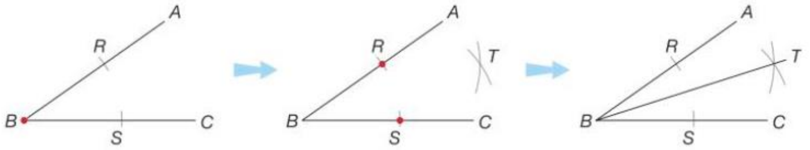
You will need a ruler and a protractor for SAS, ASA and RHS triangles.

You will need a ruler and compasses for SSS triangles.

Angle Bisector

You can use a straight edge and compasses to construct an angle bisector.

- To bisect angle ABC



- All points on the angle bisector are **equidistant** from the arms of the angle.

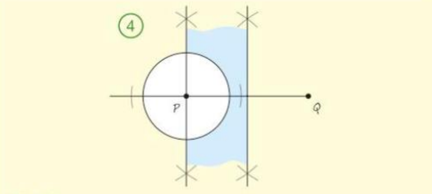
Loci Example

EXAMPLE

P and Q are two points 2.5 cm apart on a line.

Shade in the region that satisfies all these conditions:

- Right of the perpendicular to the line PQ at point P.
- Closer to P than to Q.
- More than 1 cm from P.



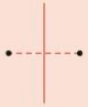
2 3 Construct the perpendicular to the line at point P. Construct the perpendicular bisector of PQ. Points to the left are nearer to P than Q.

Draw a circle radius 1 cm, centre P. Points outside are more than 1 cm from P.

Loci

- The locus of a point which is a constant distance from another point is a circle.

- The locus of a point that is **equidistant** from two other fixed points is the **perpendicular bisector** of the line joining the fixed points.

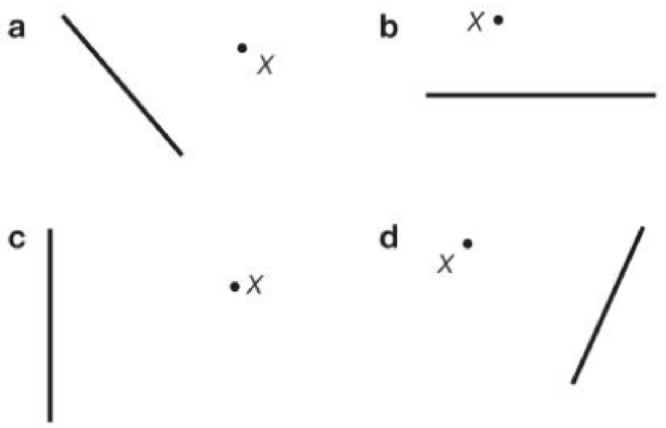


- The locus of a point at a constant distance from a fixed line is a parallel line.

- The locus of a point equidistant from two intersecting lines is the angle bisector of the lines.

GCSE Mathematics Higher – Topic 19 Constructions

Perpendicular Lines



Constructing Triangles

Use a straight edge and compasses or a protractor to construct these triangles.

- **a** Sides 8 cm, 4 cm, 7 cm (SSS)
- **b** 3 cm, 30°, 4 cm (SAS)
- **c** Sides 10 cm, 7.5 cm, 6 cm (SSS)
- **d** 8 cm, 2 cm, 90° (RHS)
- **e** Sides 6 cm, 9 cm, 5 cm (SSS)
- **f** 45°, 4 cm, 45° (ASA)

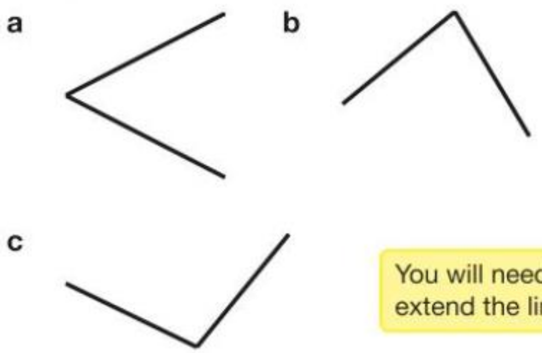
It helps to draw a rough sketch first.

Loci Exam Style 2

A lifeboat *L* is 10 km from another lifeboat *K* on a bearing of 045°. They both receive a distress call from a ship. The ship is within 7 km of *K* and within 5 km of *L*.

Draw a scale drawing to show the positions of *K* and *L*. Shade on your diagram the area in which the ship could be.

Angle Bisectors



You will need to extend the lines.

Loci Exam Style 1

The diagram shows the rectangular garden of a house.

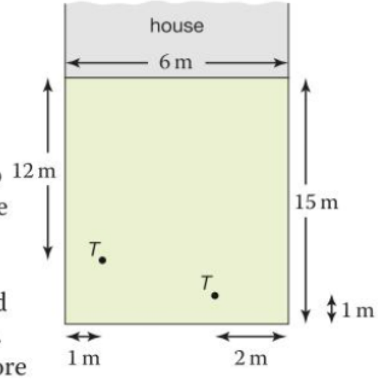
There are two trees, *T*, in the garden.

A radio mast is to be placed in the garden.

It must be more than 5 m from the rear of the house.

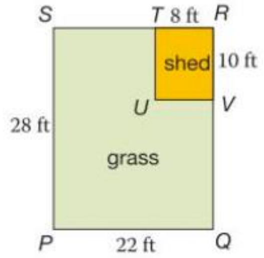
It must be more than 3 m from any tree.

Using a scale of 1 cm : 2 m, draw a scale diagram and shade the possible site for the radio mast.



Loci Exam Style 3

Pat wants to tether her goat on this grass using a 5 foot long chain. One end of the chain will be attached to the goat and the other end to a ring that can slide along an 18 foot long rail.



- **a** Calculate the area of the grass that the goat can reach if Pat puts the rail along the sides *TU* and *UV* of the shed.
- **b** Where on the perimeter of the grass should Pat put the rail to allow the goat to reach the greatest area of grass? Explain your answer.

GCSE Mathematics Higher - Topic 20 Equations 3

Key words and definitions

Quadratic: An expression where the highest order term is x^2

Substitution: Replacing a letter in an equation with a number or expression.

Elimination: A method of solving simultaneous equations that involves adding or subtracting to get rid of one of the letters.

Simultaneous: Two equations that are both satisfied by the same values.

Inequality: A relation that compares the size of two expressions.

Rearrange: Write an equation in a different way.

Solving Simultaneous Equations: Elimination

$3x + 2y = 16$, $x - 6y = 2$ → Obtain equal numbers of x's by multiplying the second equation by 3. → $3x + 2y = 16$ (1), $3x - 18y = 6$ (2) → Subtract (1) – (2) to eliminate x. → $20y = 10$, $y = \frac{1}{2}$ → Substitute for y in (1) to find x. → $3x + 1 = 16$, $x = 5$

EXAMPLE

Solve the simultaneous equations $2x - 4y = 8$, $3x + 3y = -15$

$2x - 4y = 8$ (1) multiply by 3 $6x - 12y = 24$ (3)

$3x + 3y = -15$ (2) multiply by 4 $12x + 12y = -60$ (4)

Add (3) + (4): $18x = -36$

$x = -2$

Substituting in (2): $-6 + 3y = -15$ so $3y = -9$

$y = -3$

Check the solution by substituting in one of the original equations.

Solving Simultaneous Equations: Substitution

- You can use substitution to solve **simultaneous** equations where one is **linear** and one **quadratic**.

- Rearrange the linear equation to make one unknown the subject.
- Then substitute this expression into the quadratic equation and solve.

A linear equation contains no square or higher terms. A quadratic equation contains a square term, but no higher powers.

EXAMPLE

Solve the simultaneous equations

$x + y = 7$

$x^2 + y = 13$

$x + y = 7$ (1)

$x^2 + y = 13$ (2)

Equation (1) is linear. Rearrange it to make y the subject: $y = 7 - x$

Substitute in (2): $x^2 + (7 - x) = 13$

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

Either $x = -2$ and $y = 7 - (-2) = 9$

or $x = 3$ and $y = 7 - 3 = 4$

When to Use Each Method:

Use elimination when both equations are linear.

Use substitution when one of the equations is quadratic.

Solving Linear Inequalities

- You can solve an inequality by rearranging and using inverse operations, in a similar way to solving an equation.
- If you multiply or divide an inequality by a negative number you need to reverse the inequality sign to keep it true.

$4 < 6$ but $-2 > -3$

$5 > 2$ but $-15 < -6$

EXAMPLE

a Find the range of values of x that satisfies both $3x \geqslant 2(x - 1)$ and $12 - 3x > 6$.
Represent the solution set on a number line.

b List the **integer** values of x that satisfy both inequalities.

a $3x \geq 2(x - 1)$ | $12 - 3x > 6$

$3x \geq 2x - 2$ | $12 > 6 + 3x$

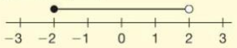
$x \geq -2$ | $6 > 3x$

$2 > x$

Combine the two inequalities.

$1\frac{1}{3} > x$ is the same as $x < 1\frac{1}{3}$.

So $-2 \leq x < 2$.

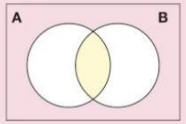


b The integer values of x that satisfy both inequalities are −2, −1, 0 and 1.

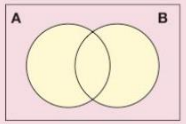
Use an 'empty' circle for < and >. Use a 'filled' circle for ≤ and ≥.

Set Notation

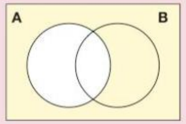
- The **intersection** of two sets, A ∩ B, consists of the elements common to both sets.



- The **union** of two sets, A ∪ B, consists of the elements which appear in at least one of the sets.



- The **complement** of a set, A', consists of the elements which are not in A.



Solving Quadratic Inequalities

EXAMPLE

Solve the quadratic inequality $x^2 + x - 6 \geqslant 0$

Replace the inequality by an equation and find its roots.

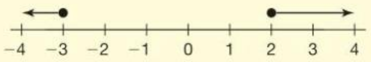
$x^2 + x - 6 = 0 \Rightarrow (x + 3)(x - 2) = 0 \Rightarrow x = -3$ or $x = 2$

Pick values of x on either side of the roots.

$x = -4$ $(-4)^2 + -4 - 6 = 6 \geq 0$

$x = 0$ $0^2 + 0 - 6 = -6 < 0$ ✗

$x = 3$ $3^2 + 3 - 6 = 6 \geq 0$ ✓



$x \leq -3$ and $x \geq 2$

You could write the solution using set notation as $\{x \mid x \leqslant -3$ or $x \geqslant 2\}$.

GCSE Mathematics Higher – Topic 20 Equations 3

Elimination

a $2x + y = 8$, $5x + 3y = 12$

b $3x + 2y = 19$, $4x - y = 29$

c $8a - 3b = 30$, $3a + b = 7$

d $2v + 3w = 12$, $5v + 4w = 23$

e $9p + 5q = 15$, $3p - 2q = -6$

f $3x - 2y = 11$, $2x - y = 8$

Harder Elimination

a $\frac{x}{3} - \frac{y}{4} = \frac{3}{2}$, $2x + y = 14$

b $\frac{a}{2} + 3b = 1$, $5a - 7b = 47$

c $p - \frac{2q}{3} = \frac{26}{3}$, $\frac{p}{4} + 3q + 1 = 0$

d $\frac{5x}{6} + \frac{y}{4} = 8$, $\frac{2x}{5} + \frac{y}{10} = 4$

Solving Linear Inequalities

a $3x \leqslant 21$

b $2x - 5 > 17$

c $\frac{p}{2} + 6 \leqslant -2$

d $28 < 7x + 49$

e $5y + 3 \leqslant 2y + 5$

f $-3y > 9$

g $4(x + 2) \leqslant 16$

h $-6x < 30$

i $\frac{x}{-5} \geqslant -2$

j $4p - 3 \leqslant 3(p - 2)$

k $3(x - 2) < 5(x + 6)$

l $6x - 4 \geqslant -2x$

Substitution

a $x^2 + y = 55$, $y = 6$

b $x + y^2 = 32$, $x = 7$

c $x^2 - 3y = 73$, $y = 9$

d $2y^2 - x = 13$, $x = 5$

Harder Substitution

a $y = x^2 - 2x$, $y = x + 4$

b $y = x^2 - 1$, $y = 2x - 2$

c $x = 2y^2$, $x = 9y - 4$

d $x = y^2 - 4$, $x = 2x - 1$

Solving Quadratic Inequalities

a $x^2 < 64$

b $x^2 > 1$

c $x^2 + 2x > 0$

d $x^2 - 6x \leqslant 0$

e $x^2 + 6x + 8 < 0$

f $x^2 + x < 12$

g $2x^2 - 5x - 3 \leqslant 0$

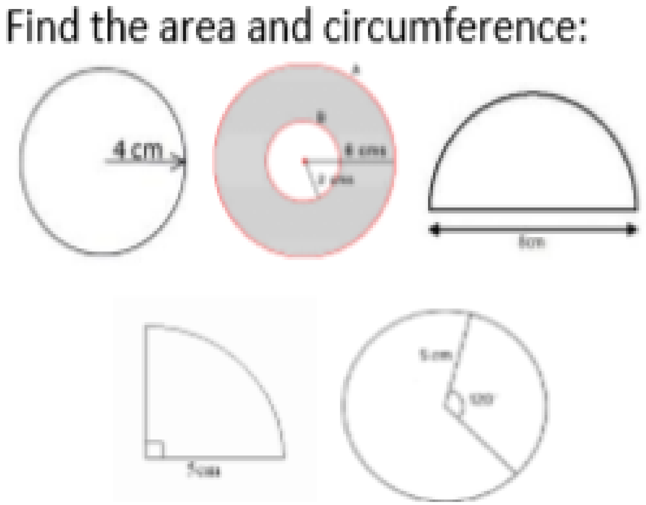
h $3x^2 + 2 \leqslant 0$

GCSE Higher Topic 21 Circle Theorems Student Knowledge Organiser

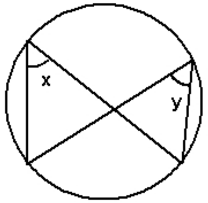
Key words and definitions

Circumference – the distance all the way around a circle
Radius – distance from centre to circumference
Diameter – the distance across the circle passing through the centre
Chord – a line connecting two points on the circumference of a circle
Segment – the area between a chord and the circumference
Tangent – a line that touches a circle
Sector – part of a circle - the area between two radiuses and the connecting arc of a circle.
Arc – part of the circumference
Perpendicular – two lines that make a right angle
Cyclic Quadrilateral - A quadrilateral with every vertex (corner point) on a circle's circumference
Semi-Circle – half a circle

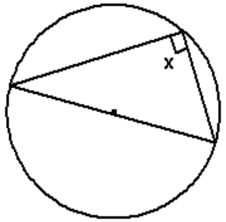
Circle Calculations



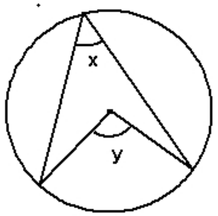
Circle Theorems



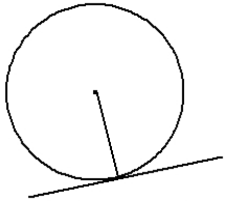
x=y (Angles at the circumference are equal)



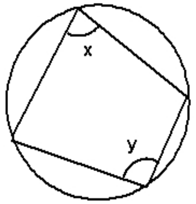
x =90 (Angle in a semicircle)



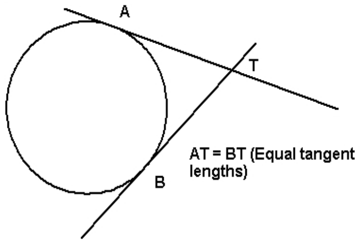
y = 2x angle (Angle at centre is twice at circumference)



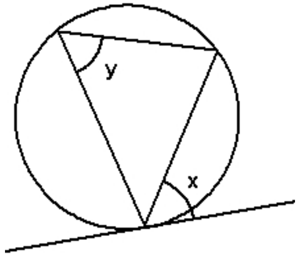
Angle between radius and tangent is 90



x+y=180 (Opposite angles of a cyclic quadrilateral add to 180)

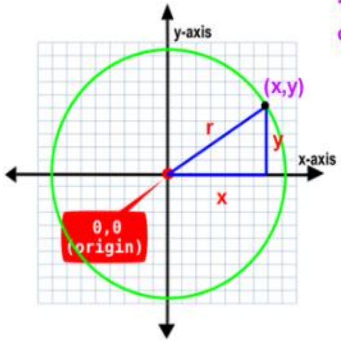


AT = BT (Equal tangent lengths)



x=y (Alternate Segment Theorem)

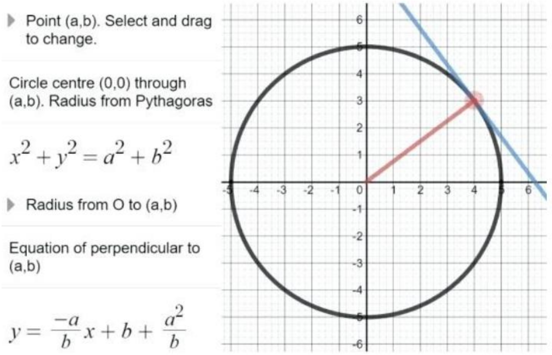
The Equation of a Circle



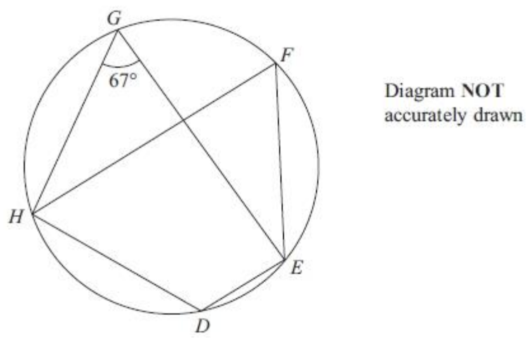
The equation of a circle centered at the origin

$x^2 + y^2 = r^2$

The Equation of a tangent to a Circle



GCSE Higher Topic 21 Circle Theorems Student Knowledge Organiser



D, *E*, *F*, *G* and *H* are points on a circle.
Angle *EGH* = 67°

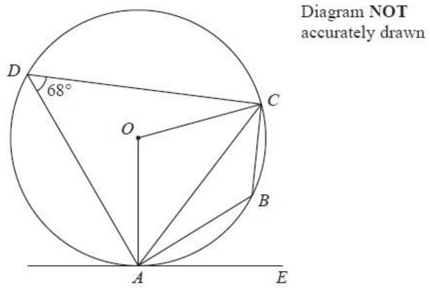
(*a*) Find the size of angle *EFH*.

………………………………………………….°

(*b*) Give a reason for your answer.

………………………………………………………………………………

………………………………………………………………………………



A, *B*, *C* and *D* are points on a circle, centre *O*.
AE is a tangent to the circle.
Angle *ADC* = 68°

(*a*) (i) Find the size of angle *ABC*.

…………………………………………………. °

(ii) Give a reason for your answer.

……………………………………………………………………………………………

b) (i) Find the size of angle *AOC*.

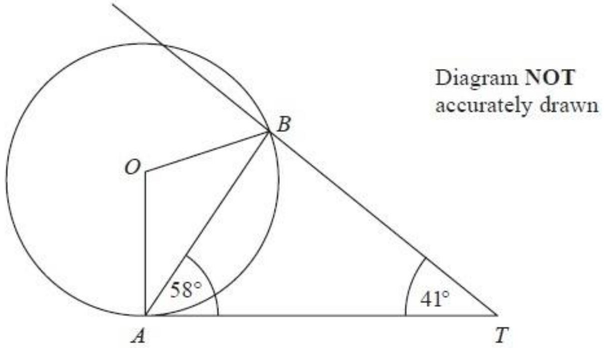
…………………………………………………. °

(ii) Give a reason for your answer.

……………………………………………………………………………………………

(*c*) Find the size of angle *CAE*

…………………………………………………. °



A and *B* are points on the circumference of a circle, centre *O*.

AT is a tangent to the circle.
Angle *TAB* = 58°.
Angle *BTA* = 41°.

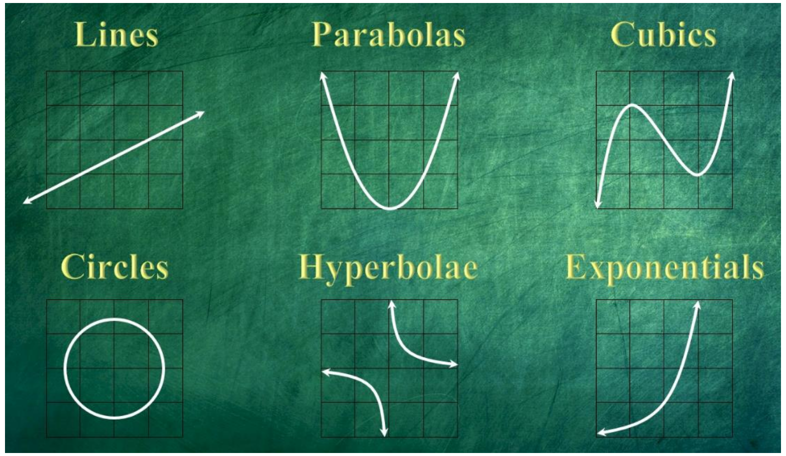
Calculate the size of angle *OBT*.
You must give reasons at each stage of your working.

GCSE Higher Topic 22 Graphs 2 Student Knowledge Organiser

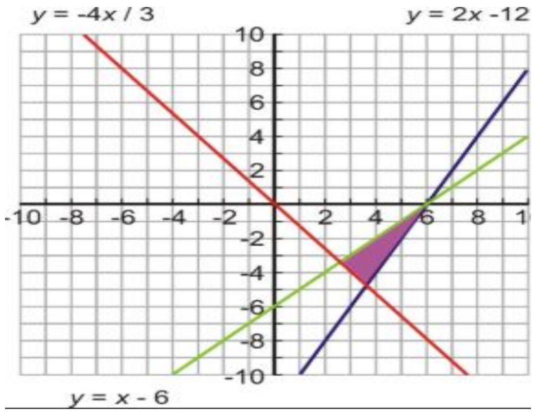
Key words and definitions

Linear Graph – A straight line y = mx + c
Quadratic Graph – Parabolic in shape y = ax² +bx + c
Cubic Graph – See Recognising Graphs y = ax³ +bx² cx + d
Exponential – See Recognising Graphs
Reciprocal – Hyperbola
Tangent – a line that touches a curve
Roots – where curve crosses the x axis
Turning Point – a point where the gradient changes direction
Maximum/Minimum – specific turning points

Recognising Graphs



Linear Graphical Inequalities



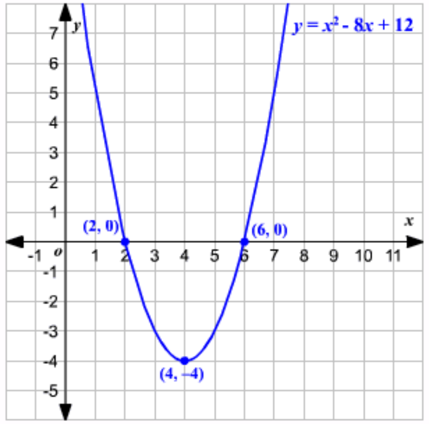
Area defined as

y > -4x/3

y > 2x - 12

y < x - 6

Quadratic Graphs



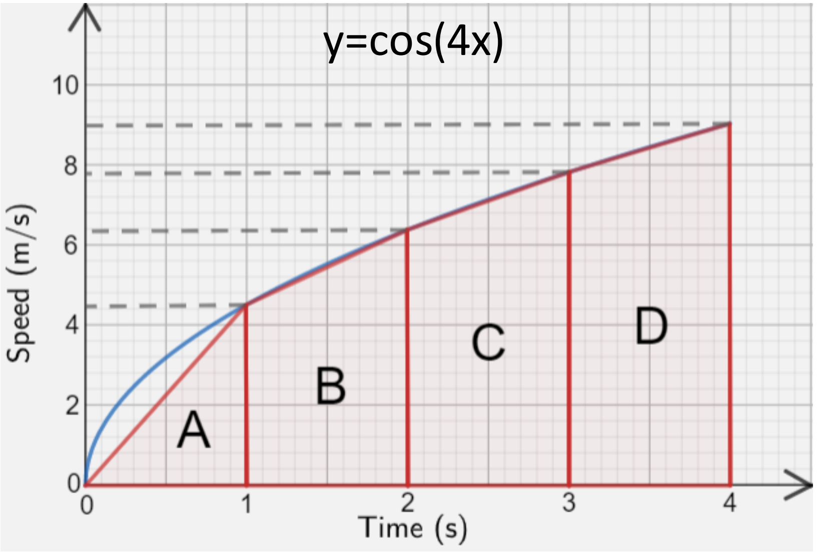
Roots

x = 2
x = 6

Turning Point

(4 , -4)

Area Under Graphs



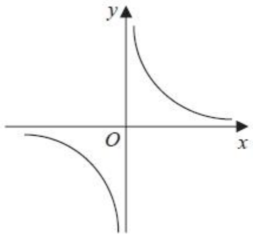
Gradients

Transforming Graphs

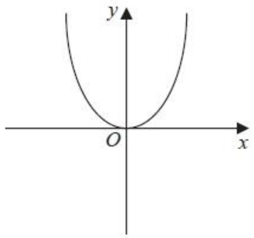
Transformation Rules for Functions		
Function Notation	Type of Transformation	Change to Coordinate Point
f(x) + d	Vertical translation up d units	(x, y) → (x, y + d)
f(x) – d	Vertical translation down d units	(x, y) → (x, y – d)
f(x + c)	Horizontal translation left c units	(x, y) → (x – c, y)
f(x – c)	Horizontal translation right c units	(x, y) → (x + c, y)
–f(x)	Reflection over x-axis	(x, y) → (x, –y)
f(–x)	Reflection over y-axis	(x, y) → (–x, y)

GCSE Higher Topic 22 Graphs 2 Student Knowledge Organiser

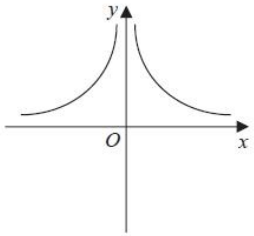
These graphs show four different proportionality relationships between y and x.



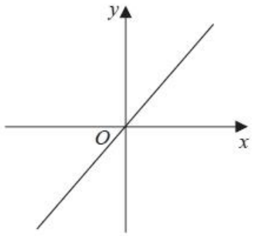
Graph A



Graph B



Graph C



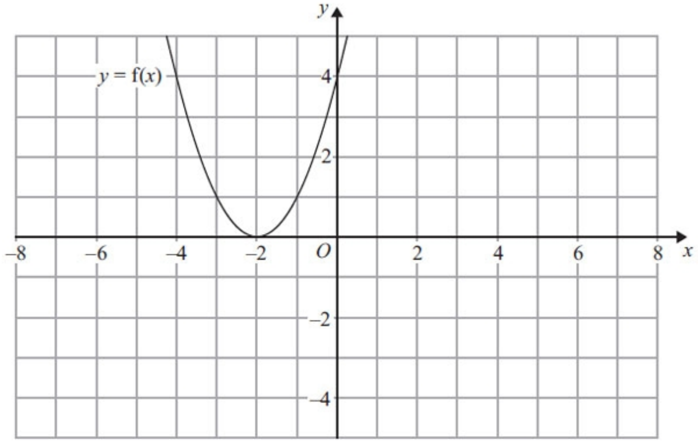
Graph D

Match each graph with a statement in the table below.

Proportionality relationship	**Graph letter**
y is directly proportional to x	
y is inversely proportional to x	
y is proportional to the square of x	
y is inversely proportional to the square of x	

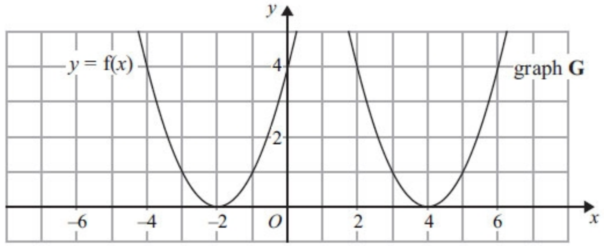
$y = f(x)$

The graph of $y = f(x)$ is shown on the grid.



(a) On the grid above, sketch the graph of $y = – f(x)$.

The graph of $y = f(x)$ is shown on the grid.

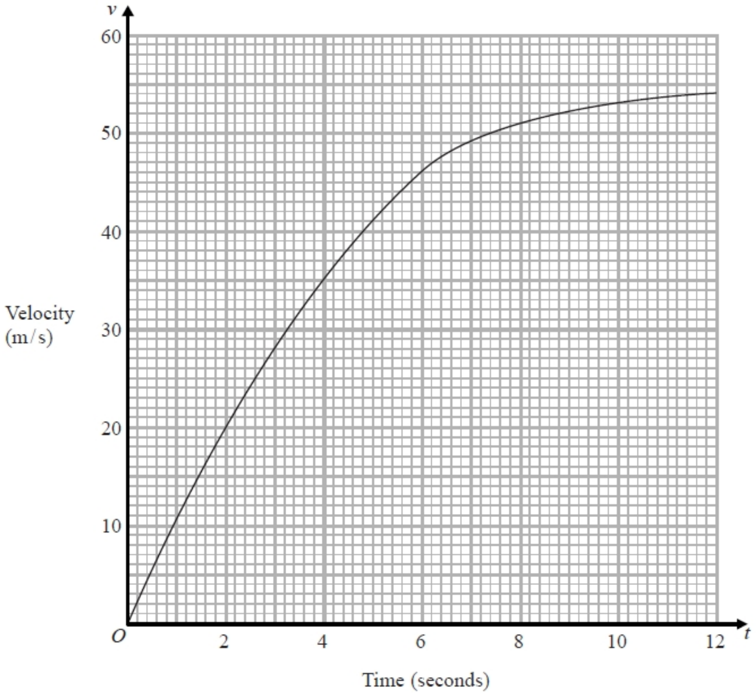


The graph **G** is a translation of the graph of $y = f(x)$.

(b) Write down the equation of graph **G**.

....

The graph shows information about the velocity, v m/s, of a parachutist t seconds after leaving a plane.



(a) Work out an estimate for the acceleration of the parachutist at $t = 6$

.... m/s^2

(b) Work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane.
Use 3 strips of equal width.

.... m

GCSE Higher Topic 23 Further Trigonometry Student Knowledge Organiser

Key words and definitions

Adjacent – the side next to the given angle in a right angled triangle
Opposite – the side opposite to the given angle in a right angled triangle
Hypotenuse – longest side of a right angled triangle
Tangent (tan) - the trigonometric ratio using Opposite and Adjacent
Cosine (cos) - the trigonometric ratio using Adjacent and Hypotenuse
Sine (sin) – the trigonometric ratio using Opposite and Hypotenuse
Perpendicular – Making a right angle
Inverse function – is a function that "reverses" another function
2D – 2 Dimensional
3D – 3 Dimenesional

Exact Values

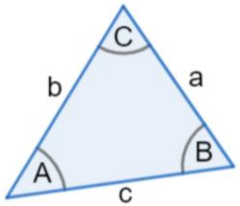
Angle (θ)	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$0°$	0	1	0
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	1	0	Not Defined

Sine and Cosine Rule

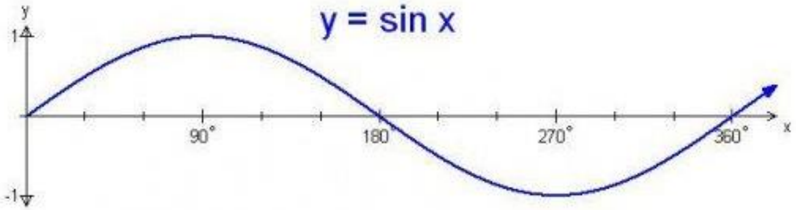
Sine Rule $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

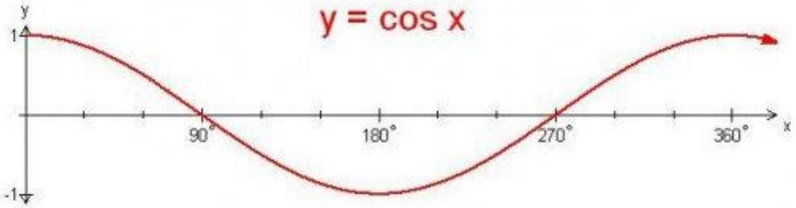
Cosine Rule $a^2 = b^2 + c^2 - 2bccosA$

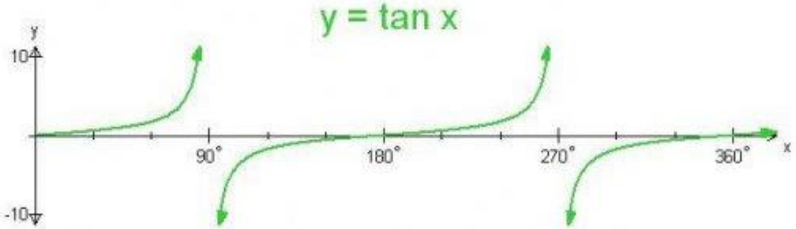
Area of a triangle $= \frac{1}{2} ab\ sinC$



Trigonometric Graphs







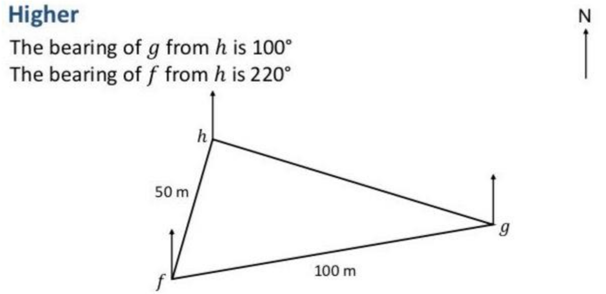
Transforming Graphs

Transform graph of y = sin (x) :
$y = -sin(x)$,
$y = sin(-x)$,
$y = sin(x) + a$,
$y = sin(x + a)$

Transform graph of y = cos (x) :
$y = -cos(x)$,
$y = cos(-x)$,
$y = cos(x) + a$,
$y = cos(x + a)$

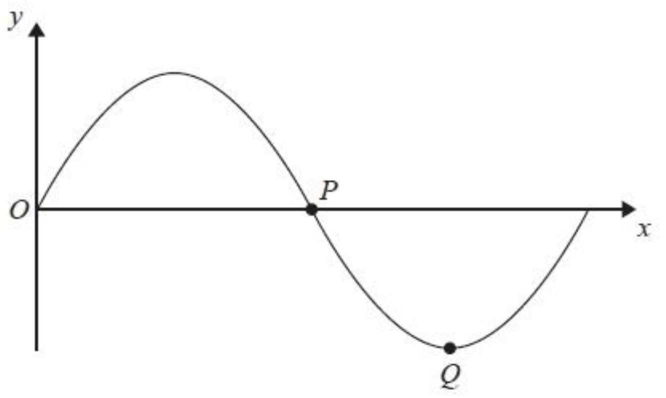
Transform graph of y = tan (x) :
$y = -tan(x)$,
$y = tan(-x)$,
$y = tan(x) + a$,
$y = tan(x + a)$

Bearings



GCSE Higher Topic 23 Further trigonometry Student Knowledge Organiser

The diagram shows part of a sketch of the curve $y = \sin x°$

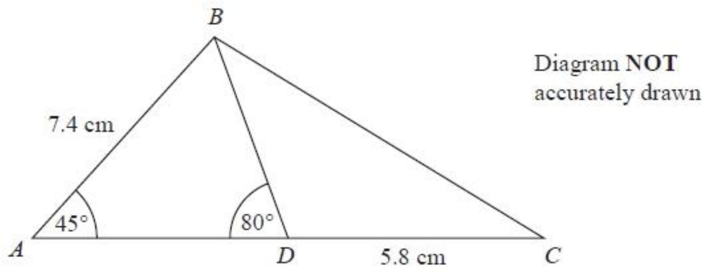


(a) Write down the coordinates of the point P.

(............................. , ...)

(b) Write down the coordinates of the point Q.

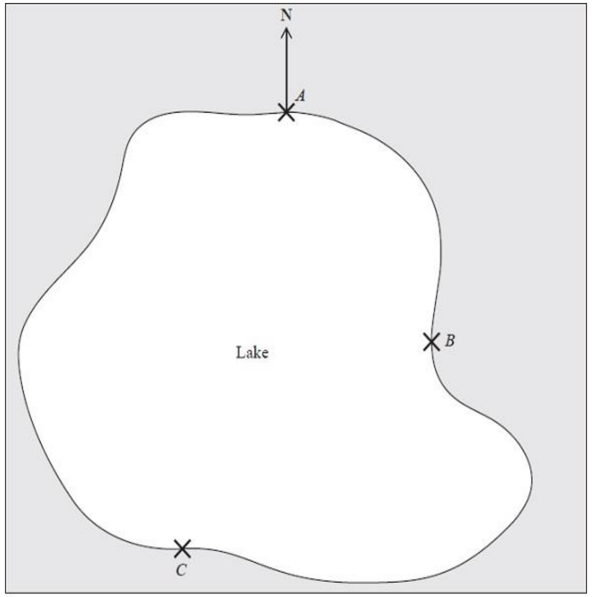
(............................. , ...)



ABC is a triangle.
D is a point on AC.
Angle $BAD = 45°$
Angle $ADB = 80°$
$AB = 7.4$ cm
$DC = 5.8$ cm

Work out the length of BC.
Give your answer correct to 3 significant figures.

The map shows the positions of three places A, B and C on the edge of a lake.



Scale 1 cm represents 2 km

(a) Find the bearing of B from A.

.... °

(1)

A ferry travels in a straight line from A to B.
It then travels in a straight line from B to C.
A speedboat travels in a straight line from A to C.

(b) How many more kilometres does the ferry travel than the speedboat?
You must show your working.

.... km

GCSE Higher Topic 24 Functions Student Knowledge Organiser

Key words and definitions

Substitution – putting values into a function to replace the variable x
Function notation – written as F(x) =
Variables – the letters in volved in the expression usually x or y
Domain – the numbers that are substituted into the function (input)
Range – the values that are obtain from substituting (output)
Inverse function – is a function that "reverses" another

Functions

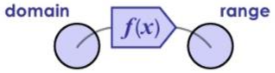
Evaluate/simplify terms like:
$f(3x)$ $f(2)$ $g(2x-3)$

NOTE

Function Domain and Range

Any function can be thought of as having an input and an output.

The 'input' is sometimes also known as the domain of the function, with the output referred to as the range.



Imporant

Each number in the domain has a unique output number in the range.

Example

The function $f(x) = x^2 + 3x$ has the domain

{ -2, -1, 0, 1, 2, 3 }

Find the range.

$f(-2) = 4 - 6 = -2$
$f(-1) = 1 - 3 = -2$
$f(0) = 0 + 0 = 0$
$f(1) = 1 + 3 = 4$
$f(2) = 4 + 6 = 10$

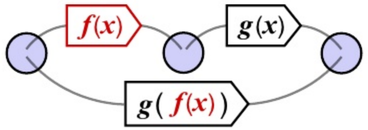
Range = { -2, 0, 4, 10 }

Composite Functions

NOTE

Composite Functions

It is possible to combine functions by substituting one function into another.



$g(f(x))$ is a composite function and is read 'g of f of x'.

Important

In general
$g(f(x)) \neq f(g(x))$

Example

Given the functions

$f(x) = 2x$

and $g(x) = x + 3$

find $f(g(x))$ and $g(f(x))$.

$f(g(x)) = 2(x+3)$
$= 2x + 6$

$g(f(x)) = (2x) + 3$
$= 2x + 3$

Composite Functions

A composite function is created when one function is substituted into another function.

Example:

Given f(x) = 3x + 2 and g(x) = x + 5

f(g(x)) = f(x+5)
= 3(x+5) +2
= 3x + 15 + 2
= 3x + 17

g(f(x)) = g(3x+2)
= (3x + 2) + 5
= 3x + 7

Inverse Functions

1. Write as an equation: $y =$
2. Swap x and y
3. Change the subject
4. Write as $f^{-1}(x) =$

A) $y = 3x - 5$
$x = 3y - 5$
$x + 5 = 3y$
$\frac{x+5}{3} = y$
$\boxed{\frac{x}{3} + \frac{5}{3} = y}$

B) $y = -64x^3$
$x = -64y^3$
$\sqrt[3]{\frac{x}{-64}} = \sqrt[3]{y^3}$
$\boxed{\frac{\sqrt[3]{x}}{-4} = y}$

GCSE Higher Topic 24 Functions Student Knowledge Organiser

Given that $f(x)=2x-4$ and $g(x)=3x+5$

a) Find: $gf(3)$

....

b) Work out an expression for: $f^{-1}(x)$

....

c) Solve: $f(x)=g(x)$

....

Given that $f(x)=3x+1$ and $g(x)=x^2$

a) Write down an expression for: $fg(x)$

....

b) Work out an expression for: $gf(x)$

....

c) Solve: $fg(x)=gf(x)$

....

A function f is defined such that

$$f(x)=x^2-1$$

Find and expression for : $f(x-2)$

....

) Hence solve: $f(x-2)=0$

....

GCSE Higher Topic 25 Proof and fractions Student Knowledge Organiser

Key words and definitions

Rationalise – to change to a rational number
Numerator - the top part of a fraction
Denominator – the bottom part of a fraction
Surd – the root of a prime number or multiple of
Rearrange – to change around using the rule of algebra

Proof

Expressions and forming expressions including
Integers - n
consecutive numbers - n , n+1 , n+2
Even numbers - 2n
Odd numbers - 2n+1
Consecutive even numbers - 2n , 2n+2 , 2n+4
Consecutive odd numbers - 2n+1 , 2n+3 , 2n+5

Change the subject of the formula

Rearranging Formulae

Make c the subject of the formula y = mx + c.

$$y = mx + c$$
$$-mx \qquad -mx$$
$$y - mx = c$$

Make m the subject of the formula y = mx + c.

$$y = mx + c$$
$$-c \qquad -c$$
$$y - c = mx$$
$$\div x \qquad \div x$$
$$\frac{y-c}{x} = m$$

Algebraic Fractions

Simplify fractions like:

$$\frac{x^2+3x-4}{2x^2-5x+3}$$

Add, subtract, multiply and divide algebraic fractions like:

$$\frac{4}{x+2}+\frac{3}{x-2}$$

Direct Proof

A proof is a logical and structured argument to show that a mathematical statement (or **conjecture**) is always true.
A mathematical proof usually starts with previously established mathematical facts (or **theorems**) and then works through a series of logical steps. The final step in a proof is a **statement** of what has been proven.

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Known facts or theorems → Clearly shown logical steps → Statement of proof

- **In a mathematical proof you must**
  - **State any information or assumptions you are using**
  - **Show every step of your proof clearly**
  - **Make sure that every step follows logically from the previous step**
  - **Make sure you have covered all possible cases**
  - **Write a statement of proof at the end of your working**

Direct Proof

Model expressions that could be a multiple of 7 (7n) or a multiple of 8 (8n) or 2 more than multiple of 3 (3n+2)

Algebraic Proof

Prove the sum of four consecutive numbers is always even.

$$x + (x + 1) + (x + 2) + x + 3)$$
$$4x + 6$$
$$2(2x + 3)$$

Prove $(n + 6)^2 - (n + 2)^2$ is always a multiple of 8

$$(n + 6)(n + 6) - [(n + 2)(n + 2)]$$
$$n^2 + 6n + 6n + 36 - [n^2 + 2n + 2n + 4]$$
$$n^2 + 12n + 36 - [n^2 + 4n + 4]$$
$$n^2 + 12n + 36 - n^2 - 4n - 4$$
$$8n + 32$$
$$8(n + 4)$$

GCSE Higher Topic 25 Proof and fractions Student Knowledge Organiser

n is an integer greater than 1.
Use algebra to show that $(n^2 - 1) + (n - 1)^2$ is always equal to an even number.

Prove that the difference between the squares of any two consecutive even numbers is always an odd number multiplied by 4.

Here are the first 4 lines of a number pattern.

$1 + 2 + 3 + 4$	$=$	$(4 \times 3) - (2 \times 1)$
$2 + 3 + 4 + 5$	$=$	$(5 \times 4) - (3 \times 2)$
$3 + 4 + 5 + 6$	$=$	$(6 \times 5) - (4 \times 3)$
$4 + 5 + 6 + 7$	$=$	$(7 \times 6) - (5 \times 4)$

n is the first number in the nth line of the number pattern.
Show that the above number pattern is true for the four consecutive integers n, $(n + 1)$, $(n + 2)$ and $(n + 3)$.

Show that

$$\frac{3x+6}{x^2-3x-10} \div \frac{x+5}{x^3-25x}$$

simplifies to ax where a is an integer

GCSE Higher Topic 26 Vectors Student Knowledge Organiser

Key words and definitions

Scalar: a number (measure) with magnitude only

Vector: an illustrative measure which has both magnitude and direction

Magnitude the length of a vector (found using Pythagoras' theorem)

Pythagoras - $a^2 + b^2 = c^2$

Direction: the angle of the vector (often found using trigonometry)

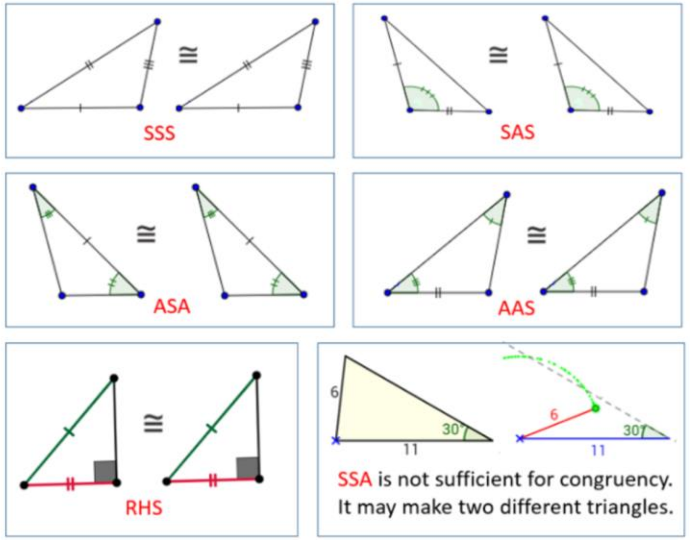
Column: 2 or 3 dimensional matrix isolating dimensional movement

Multiple - many of the same type

Parallel: vectors which are scalar multiples of one another

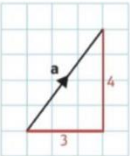
-

Congruent Triangles



Vectors

A vector can be described by its change in position or **displacement** relative to the x- and y-axes.



$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ where 3 is the change in the x-direction and 4 is the change in the y-direction.

This is called **column vector** form.

Notation The top number is the x-component and the bottom number is the y-component.

- **To multiply a column vector by a scalar, multiply each component by the scalar:** $\lambda\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \lambda p \\ \lambda q \end{pmatrix}$
- **To add two column vectors, add the x-components and the y-components:** $\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p+r \\ q+s \end{pmatrix}$

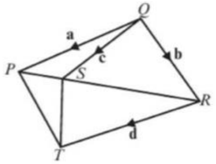
Use of vectors

In the diagram, $\overrightarrow{QP} = \mathbf{a}$, $\overrightarrow{QR} = \mathbf{b}$, $\overrightarrow{QS} = \mathbf{c}$ and $\overrightarrow{RT} = \mathbf{d}$.

Find in terms of **a**, **b**, **c** and **d**:

a $\overrightarrow{PS}$ **b** $\overrightarrow{RP}$

c $\overrightarrow{PT}$ **d** $\overrightarrow{TS}$



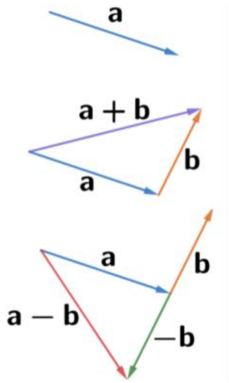
a $\overrightarrow{PS} = \overrightarrow{PQ} + \overrightarrow{QS} = -\mathbf{a} + \mathbf{c}$
$= \mathbf{c} - \mathbf{a}$
Add vectors using $\triangle PQS$.

b $\overrightarrow{RP} = \overrightarrow{RQ} + \overrightarrow{QP} = -\mathbf{b} + \mathbf{a}$
$= \mathbf{a} - \mathbf{b}$
Add vectors using $\triangle RQP$.

c $\overrightarrow{PT} = \overrightarrow{PR} + \overrightarrow{RT} = (\mathbf{b} - \mathbf{a}) + \mathbf{d}$
$= \mathbf{b} + \mathbf{d} - \mathbf{a}$
Add vectors using $\triangle PRT$.
Use $\overrightarrow{PR} = -\overrightarrow{RP} = -(\mathbf{a} - \mathbf{b}) = \mathbf{b} - \mathbf{a}$.

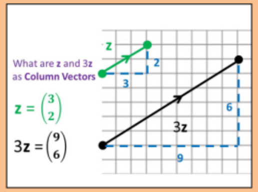
d $\overrightarrow{TS} = \overrightarrow{TR} + \overrightarrow{RS} = -\mathbf{d} + (\overrightarrow{RQ} + \overrightarrow{QS})$
$= -\mathbf{d} + (-\mathbf{b} + \mathbf{c})$
$= \mathbf{c} - \mathbf{b} - \mathbf{d}$
Add vectors using $\triangle TRS$ and $\triangle RQS$.

Vector addition and multiples of vectors



$\mathbf{z} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$3\mathbf{z} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

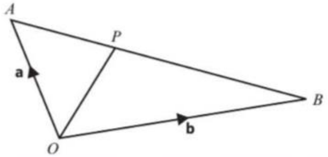


$3\mathbf{z} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{matrix} \times 3 \\ \times 3 \end{matrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

Geometric Problems - Vectors

In the diagram the points A and B have position vectors **a** and **b** respectively (referred to the origin O). The point P divides AB in the ratio 1 : 2.

Find the position vector of P.



$\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$
There are 3 parts in the ratio in total, so P is $\frac{1}{3}$ of the way along the line segment AB.

$= \overrightarrow{OA} + \frac{1}{3}(\overrightarrow{OB} - \overrightarrow{OA})$
Rewrite $\overrightarrow{AB}$ in terms of the position vectors for A and B.

$= \frac{2}{3}\overrightarrow{OA} + \frac{1}{3}\overrightarrow{OB}$

$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$
Give your final answer in terms of **a** and **b**.

GCSE Higher Topic 26 Vectors Student Knowledge Organiser

The diagram shows a regular hexagon *OABCDE*.

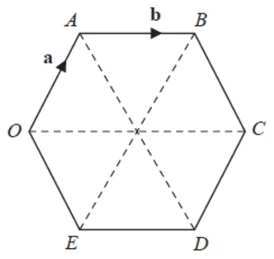


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{a}$

$\overrightarrow{AB} = \mathbf{b}$

M is the midpoint of *OE*.
N is the midpoint of *AB*.

(a) Find $\overrightarrow{MN}$ in terms of **a** and/or **b**.

$\overrightarrow{MN}$ = ...

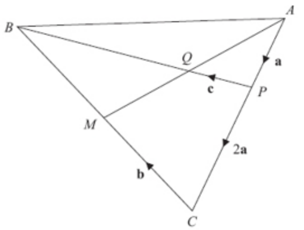


Diagram **NOT** accurately drawn

M is the midpoint of *BC*.
Q is the midpoint of *AM*.

$\overrightarrow{AP} = \mathbf{a}$ $\quad \overrightarrow{PC} = 2\mathbf{a}$ $\quad \overrightarrow{CM} = \mathbf{b}$ $\quad \overrightarrow{PQ} = \mathbf{c}$

(a) Find $\overrightarrow{AM}$ in terms of **a** and **b**.

$\overrightarrow{AM}$ = ...

(b) Find $\overrightarrow{QB}$ in terms of **c**.

$\overrightarrow{QB}$ = ...

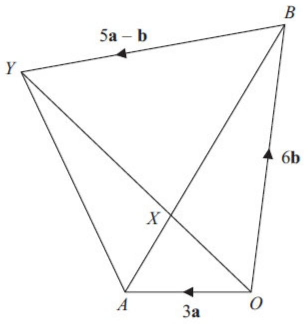


Diagram **NOT** accurately drawn

OAYB is a quadrilateral.

$\overrightarrow{OA} = 3\mathbf{a}$

$\overrightarrow{OB} = 6\mathbf{b}$

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**.

....

X is the point on *AB* such that *AX* : *XB* = 1 : 2

and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$