

Knowledge Organisers and Practice questions

- Year 9 Autumn term test covers Topics 1 – 3
- Year 9 Spring term test covers Topics 1 – 6
- Year 9 Summer term test covers Topics 1 – 12
- Year 10 Autumn term test covers Topics 1 – 15
- Year 10 Spring term test covers Topics 1 – 19
- Year 10 Summer term test covers Topics 1 – 22

Topic	Title	
1	Factors and Multiples	Types of number; <input type="radio"/>
		Written calculations; <input type="radio"/>
		Factors and multiples; <input type="radio"/>
		Negative numbers; <input type="radio"/>
		Rounding and approximations <input type="radio"/>
		Bounds <input type="radio"/>
		Indices; <input type="radio"/>
2	Number and Calculations	Standard form; <input type="radio"/>
		Order of operations; <input type="radio"/>
		Calculator methods <input type="radio"/>
		Substitution; <input type="radio"/>
		Simplifying expressions; <input type="radio"/>
		Expanding single and double brackets; <input type="radio"/>
3	Expressions 1	Factorising single and double brackets <input type="radio"/>
		Solve linear equations; <input type="radio"/>
		Form and solve linear equations; <input type="radio"/>
4	Equations 1	Change the subject <input type="radio"/>
		Sampling: random, systematic and stratified <input type="radio"/>
5	Charts and Averages	Tally, Bar, Pie and Scatter charts; <input type="radio"/>
		Averages from lists; <input type="radio"/>
		Average from tables; <input type="radio"/>
		Reverse means <input type="radio"/>

Topic	Title	
6	Area and Volume 1	Area; <input type="radio"/>
		Volume; <input type="radio"/>
		Surface area <input type="radio"/>
7	Fractions, Decimals and Percentages 1	Fractions: add, subtract, multiply and divide <input type="radio"/>
		Find fractions of an amount <input type="radio"/>
		Find percentages of an amount <input type="radio"/>
		Percentage change <input type="radio"/>
		Reverse percentages <input type="radio"/>
		FDP conversions <input type="radio"/>
		Simplifying ratio; <input type="radio"/>
8	Ratio	Splitting into a ratio; <input type="radio"/>
		Speed, density, pressure; <input type="radio"/>
		Time; <input type="radio"/>
		Ratio problems <input type="radio"/>
		Properties of quadrilaterals; <input type="radio"/>
		Angles about a point, on a straight line and in a triangle; <input type="radio"/>
		Angles on parallel lines (corresponding, alternate and vertically opposite) <input type="radio"/>
9	Shapes and Angles	
10	Pythagoras and Trigonometry	Pythagoras' Theorem; <input type="radio"/>
		Trigonometry in right angled triangles to find angles <input type="radio"/>
		Trigonometry in right angled triangles to find sides <input type="radio"/>

Topic	Title	
11	Sequences	Pattern recognition and continuation; <input type="radio"/>
		n th term of linear sequences; <input type="radio"/>
		Quadratic sequences; <input type="radio"/>
		Geometric sequences; <input type="radio"/>
		Fibonacci sequences <input type="radio"/>
12	Probability	Probability scales; <input type="radio"/>
		Simple probability; <input type="radio"/>
		Sample space; <input type="radio"/>
		Venn diagrams; <input type="radio"/>
		Tree diagrams <input type="radio"/>
13	Transformations	Translation; <input type="radio"/>
		Rotation; <input type="radio"/>
		Reflection; <input type="radio"/>
		Enlargement; <input type="radio"/>
		Plans and elevations <input type="radio"/>
14	Expressions 2	Expanding single and double brackets; <input type="radio"/>
		Factorising single and double brackets <input type="radio"/>
		Solve quadratic equations <input type="radio"/>
15	Area and Volume 2	Area incl. sectors; <input type="radio"/>
		Volume; <input type="radio"/>
		Surface area; <input type="radio"/>
		Similarity <input type="radio"/>

Topic	Title		
16	Graphs	Coordinates;	<input type="radio"/>
		Drawing straight lines;	<input type="radio"/>
		Give the equation of a straight line;	<input type="radio"/>
17	Constructions	Midpoints and line length	<input type="radio"/>
		Constructions: triangles, perpendicular bisectors, angle bisectors	<input type="radio"/>
		Locii;	<input type="radio"/>
		Bearings;	<input type="radio"/>
		Tessellations	<input type="radio"/>
		Fractions: add, subtract, multiply and divide	<input type="radio"/>
		Find fractions of an amount	<input type="radio"/>
18	Fractions , Decimals and Percentages 1	Find percentages of an amount	<input type="radio"/>
		Percentage change	<input type="radio"/>
		Reverse percentages	<input type="radio"/>
		FDP conversions	<input type="radio"/>
		Plotting quadratic graphs;	<input type="radio"/>
		Plotting other shapes;	<input type="radio"/>
		Equation of a straight line	<input type="radio"/>
19	Graphs 2	Linear equations;	<input type="radio"/>
		Simultaneous linear equations	<input type="radio"/>
		Simplifying ratio;	<input type="radio"/>
20	Equation s 2	Splitting into a ratio;	<input type="radio"/>
		Speed, density, pressure;	<input type="radio"/>
		Time;	<input type="radio"/>
		Ratio problems	<input type="radio"/>
		Geometric proof;	<input type="radio"/>
21	Ratio	Column vectors	<input type="radio"/>
		Drawing vectors	<input type="radio"/>
		Vectors around a shape	<input type="radio"/>
		Vectors	<input type="radio"/>
22	Vectors	Geometric proof;	<input type="radio"/>
		Column vectors	<input type="radio"/>
		Drawing vectors	<input type="radio"/>
		Vectors around a shape	<input type="radio"/>

GCSE Foundation Topic 1 Factors and Multiples Student Knowledge Organiser

Key words and definitions

A **factor** is a number that divides into another number exactly and without leaving a remainder.

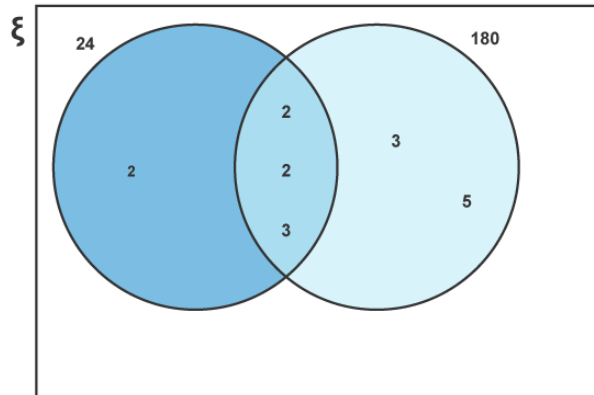
A **prime number** has only two factors - the number itself and 1.

1 is not a prime number

A **Multiple** is the result of multiplying a number by an integer. The times tables of a number.

HCF & LCM from Venn diagrams

Put each prime factor in the correct place in the Venn diagram. Any common factors should be placed in the intersection of the two circles.



The highest common factor is found by **multiplying together the numbers in the intersection** of the two circles.

$$\text{HCF} = 2 \times 2 \times 3 = 12$$

The LCM is found by **multiplying together the numbers from all three sections** of the circles.

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

Rounding to Significant Figures

Examples

Round 53,879 to 1 significant figure, then 2 significant figures.

5 | 3879 to 1 significant figure is 50,000

53 | 879 to 2 significant figures is 54,000

Notice that the number of significant figures in the question is the maximum number of non-zero digits in your answer.

Round 0.005089 to 1 significant figure, then 2 significant figures.

0.005 | 089 to 1 significant figure is 0.005

0.0050 | 89 to 2 significant figures is 0.0051

Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

HCF Example

Consider the numbers 12 and 15:

The factors of 12 are : **1, 2, 3, 4, 6, 12.**

The factors of 15 are : **1, 3, 5, 15.**

1 and 3 are the only **common factors** (numbers which are factors of both 12 and 15).

Therefore, the **highest common factor** of 12 and 15 is **3**.

LCM Example

Consider the numbers 12 and 15 again:

The multiples of 12 are : **12, 24, 36, 48, 60, 72, 84,**

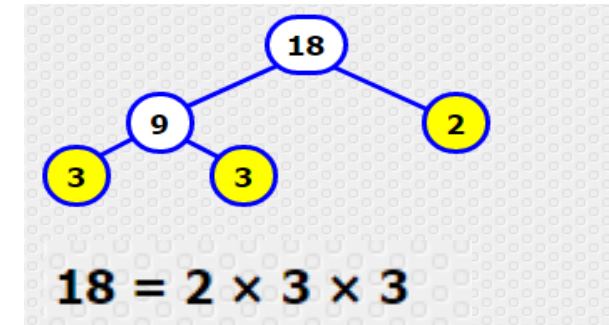
The multiples of 15 are : **15, 30, 45, 60, 75, 90,**

60 is a **common multiple** (a multiple of both 12 and 15), and there are no lower common multiples.

Therefore, the **lowest common multiple** of 12 and 15 is **60**.

Product of Prime Factors

Finding out which prime numbers multiply together to make the original number. Use a prime factor tree. Also known as 'prime factorisation'.



Error Intervals

An error interval is the range of values that a number could have taken before being rounded or truncated. Error intervals are usually written as a range using inequalities, with a lower bound and an upper bound.

Write down the error interval for y.

$$4.13 \text{ 2dp } \begin{matrix} \nearrow 0.005 \rightarrow 4.135 \\ \searrow 0.005 \rightarrow 4.125 \end{matrix}$$

(b)(i) $4.125 \leq y < 4.135$

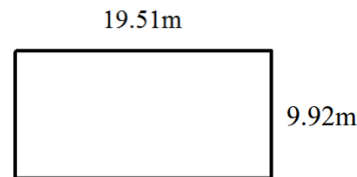
Product of Prime Factors

Write the following as the product of their prime factors

- (a) 70
- (b) 90
- (c) 24
- (d) 126
- (e) 75
- (f) 84
- (g) 99
- (h) 500

Estimation

A diagram of a farmer's field is shown below:



- a. Calculate an estimate for the area of the field.
- b. Each cow needs at least 4.5m^2 of room in a field, use your answer to part a to estimate the maximum number of cows which can be placed in the field.

HCF & LCM

1. Find the Highest Common Factor of these numbers:

- (a) 18 and 30
- (b) 15 and 20
- (c) 16 and 24
- (d) 12 and 36
- (e) 28 and 70
- (f) 39 and 65
- (g) 38 and 57
- (h) 20 and 30

2. Find the Lowest Common Multiple of these numbers

- (a) 6 and 7
- (b) 4 and 6
- (c) 5 and 8
- (d) 10 and 4
- (e) 16 and 5
- (f) 14 and 21
- (g) 2.2 and 5
- (h) 0.4 and 7

3. The lowest common multiple of two numbers is 36, one number is 12, what might the other number be?

4. Jack thinks of two numbers, the HCF of these numbers is 6 and one of the numbers is 24 suggest what his other number may have been.

Error Intervals

Write down the error interval for each of the following questions.

- 1:** The number of passengers on a coach, g , rounded to the nearest 10 is 70 people. Write down the error interval for g
- 2:** A number, g , rounded to the nearest whole number is 241. Write down the error interval for g
- 3:** The density of an alloy, m , correct to 2 significant figures is 5.9g/cm^3 . Write down the error interval for m
- 4:** A number, p , **truncated** to 2 decimal places is 13.19. Write down the error interval for p
- 5:** The weight of a pencil case, w , rounded to the nearest 100g is 900g. Write down the error interval for w
- 6:** The length of a piece of string, j , rounded to 1 decimal place is 48.2cm. Write down the error interval for the length j
- 7:** The volume of a box, d , correct to 1 significant figure is 70cm^3 . Write down the error interval for d
- 8:** The weight of a suitcase, u , **truncated** to 1 decimal place is 13.2kg. Write down the error interval for the weight of the suitcase.
- 9:** A number, r , rounded to 2 decimal places is 4.05. Write down the error interval for r
- 10:** A number, k , correct to 3 significant figures is 4.45. Write down the error interval for k

Key words and definitions

Index number - number that is multiplied by itself one or more times is raised to a power. The power is the index number. The plural is indices.

Power - A number that is multiplied by itself one or more times is raised to a power

Standard Form – Writing large and small numbers as a number between 1 and 10 multiplied by a power of 10

Square Root – square root of a number is a value that, when multiplied by itself, gives the number

Powers/Indices

2^4 is a short way of writing $2 \times 2 \times 2 \times 2$.

Index laws

1. $a^m a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $(ab)^m = a^m b^m$

5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

6. $a^0 = 1$

7. $a^{-n} = \frac{1}{a^n}$

e.g. $b^5 \times b^3 = b^{5+3} = b^8$

e.g. $(a^2)^5 = a^{2 \times 5} = a^{10}$

Standard Form

Standard form, or standard index form, is a system of writing numbers which can be very large or very small numbers. It is based on using powers of 10.

Convert to
50,000 can be written as: $5 \times 10,000$

$$10,000 = 10 \times 10 \times 10 \times 10 = 10^4$$

$$\text{So, } 50,000 = 5 \times 10^4$$

0.0005 can be written as 5×0.0001 .

$$0.0001 = 10^{-4}$$

$$\text{So, } 0.0005 = 5 \times 10^{-4}.$$

Convert from

1.34×10^3 is 1,340, since $1.34 \times 10 \times 10 \times 10 = 1,340$.

4.78×10^{-3} is 0.00478, as $4.78 \times 0.001 = 0.00478$.

BIDMAS

Mathematical operations must be carried out in the correct order. BIDMAS is a way of remembering this order.

B – Brackets

I – Indices/Powers

D – Division

M – Multiplication

A – Addition

S – Subtraction

e.g. $2^2 \times 5 - 6 \div 3$.

1. There are no brackets (B), so calculate the indices first (I), giving

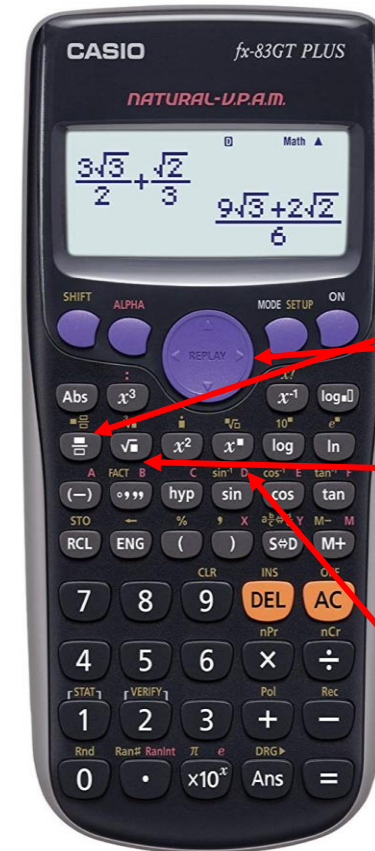
$$4 \times 5 - 6 \div 3$$

2. Do any divisions or multiplications (DM), working left to right:

$$4 \times 5 = 20 \text{ and } 6 \div 3 = 2$$

3. And, finally, do any additions or subtractions (AS): $20 - 2$ gives 18

Calculator Methods



Calculate

$$\frac{\sqrt{678} - 1.42^2}{3^4 \times 1.57}$$

1. Press fraction button (use arrow keys to move around the fraction)
2. Press square root button and fill in 678, use arrow key to go out of the square root
3. Subtract 1.42^2
4. Move down the fraction and press the indices button and use the arrow keys to fill in the numbers and multiply by 1.57

GCSE Foundation Topic 2 Number and calculations Student Knowledge Organiser

Index Laws

(h) $2^8 \times 2^8$ (i) $2^9 \times 2^2$ (j) 2×2^8 (k) $2^6 \times 2^5$

(a) $5^5 \div 5^2$ (b) $5^8 \div 5^3$ (c) $5^9 \div 5^2$ (d) $5^7 \div 5^5$

(a) $(8^5)^2$ (b) $(8^3)^2$ (c) $(8^4)^3$

$(2a)^4$ $(ab^2)^3$ $(3x^4)^2$

2^{-4} b^{-3} 3^{-2}

Bidmas

(a) $7 + 2 \times 3$ (b) $9 + 4 \times 2$ (c) $10 + 2 \times 2$

(a) $5 - 2^2$ (b) $7 + 3^2$ (g) $(1 + 2)^3$

(e) $8 + (5 - 1) \times 3$ (f) $50 - (1 + 4) \times 4$ (g) $19 \times 2 + 5^2$

(a) $5 \times 3 + 2 \times 6$ (b) $9 \div 3 + 15 \times 2$ (c) $10 \div 2 - 2 \times 1$

(i) $10 - \sqrt{16}$ (j) $\sqrt{2 + 14}$ (k) $\sqrt{4 + 3^2}$

Standard Form

Convert to standard form

(e) 100000000 (f) 900 (g) 250000

(i) 54000000 (j) 11000000 (k) 89000

(e) 0.00065 (f) 0.0022 (g) 0.0361

(i) 0.00000423 (j) 0.0000000981 (k) 0.00407

Convert from standard form

(e) 5×10^7 (f) 1.2×10^2 (g) 2.9×10^5

(i) 3.16×10^{-5} (j) 8.62×10^{-4} (k) 7.09×10^{-6}

Calculator Methods

Use your calculator to find the exact value to these calculations

(a) $\frac{3.5}{1.4 + 3.8}$ (b) $\frac{7.8 \times 5.3}{11.7}$

(d) $\frac{0.18 + 0.175}{2.4 \times 1.9}$ (e) $\frac{0.495}{0.091} \times 604.6$

(d) $\frac{18.2 + 7.4}{\sqrt{9.22}}$ (e) $\frac{\sqrt{17.8 - 9.93}}{1.25 - 5.9}$

Applying Knowledge

Hannah thinks the answer to should be 4 because $3.25 - 1.25^2 = 2$ and then $2^2 = 4$ Explain why the answer on her calculator is 1.6875.

Put brackets in the following statements to make them true

(a) $6 \times 7 + 3 - 8 = 52$

(b) $4 + 3 \times 7 - 1 = 42$

Show that 5.9×10^8 is approximately 100 times bigger than 4.2×10^6

Simplify

(i) $y^6 \times y^5 \times y^2$ (j) $y^8 \times y \times y^3$

$2a^3c^3 \times 3a^2c$

Key words and definitions

Substitution: putting numbers where the letters are

Simplify: make an algebraic expression easily understandable and solvable

Expand: multiply to remove the ()

Factorise: putting an expression back into brackets.

Substitution

$$2b^2c = 2 \times b^2 \times c$$

(substituting $b = 4$ $c = 3$)

This gives: $2b^2c = 2 \times b^2 \times c = 2 \times 4^2 \times 3$

$$2 \times 16 \times 3 = 96$$

Simplify

Simplify $b + b + b + b$.

Adding the four like terms together gives $4b$.

Simplify $5m + 3m - 2m = 8m - 2m = 6m$

Expand (Single Brackets)

To remove brackets, multiply the term on the outside of the bracket with each term inside the brackets.

Expand and simplify:

$$\begin{aligned} 3(x + y) + 2(x + y) &= 3x + 3y + 2x + 2y \\ &= 5x + 5y \end{aligned}$$

Simplify (with multiplication and division)

Simplify $b \times b \times b$.

$$b \times b \times b = b^3.$$

Simplify $16e^2 \div 2e$.

$$16 \div 2 = 8 \text{ and } e^2 \div e = e$$

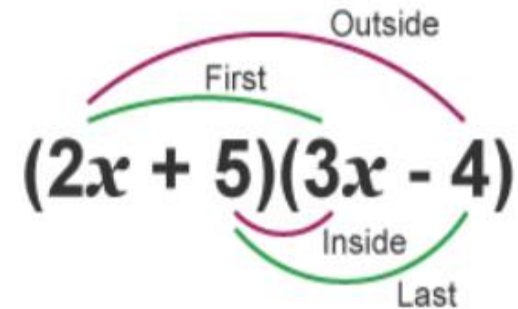
$16e^2 \div 2e$ simplified is $8e$.

When you divide powers you subtract them (shown below):

$$\frac{a^5}{a^2} = a^{5-2} = a^3$$

Expand (Double Brackets)

Expand and Simplify $(2x + 5)(3x - 4)$



$$\begin{aligned} &(2x + 5)(3x - 4) \\ &= (2x \times 3x) + (2x \times -4) + (5 \times 3x) + (5 \times -4) \\ &= 6x^2 - 8x + 15x - 20 \\ &= 6x^2 + 7x - 20 \end{aligned}$$

Simplify:

1) $3a + 2b + 4c + 2a$

2) $5c + 2d + 3c + 4d$

3) $2b - b + 4a - 3c$

4) $5a + 2b + 6a - 2b$

5) $8h - 4g + 5g + 2h =$

6) $7b + 6a - 5b + 3a =$

7) $5k + 4j - 3k + 6j =$

8) $3a + 2a - 5a + b =$

Expand:

1) $5(a + 3)$

2) $3(b + 4)$

3) $6(c - 2)$

4) $4(d - 5)$

5) $3(2e + 4)$

6) $7(6f - 2)$

7) $8(3 - 2g)$

8) $9(7 + 4h)$

$6(x + 3) =$

$7(x + 7) =$

$11(x + 1) =$

$9(x + 8) =$

$4(x + 7) =$

1) $(x + 4)(x - 2)$

2) $(x + 6)(x + 3)$

3) $(x - 7)(x - 9)$

4) $(x - 2)(x - 8)$

5) $(x - 4)(x + 6)$

1) $(2x + 4)(x - 7)$

2) $(x + 2)(3x + 3)$

3) $(4x - 6)(x - 9)$

4) $(5x - 6)(2x + 3)$

Substitution: Find the value of

Let $x = 4$

and $y = -3$

1) $3x + 6$

2) $3x - 15$

3) $2y - 2$

4) $4x + 9$

5) $3 - x$

6) $3 - y$

7) $5 - 2x$

Let $s = -2$

and $t = -4$

1) $s^2 + 4$

2) $s^2 - 7$

3) $2s^2 + 5$

4) $6s^2 - 2$

5) $t^2 + 7$

6) $t^2 - 20$

7) $t^3 + 1$

GCSE Foundation Topic 4 Equations 1 Student Knowledge Organiser

Key words and definitions

Variable – A symbol for an unknown value. Usually a letter, such as a , x or y

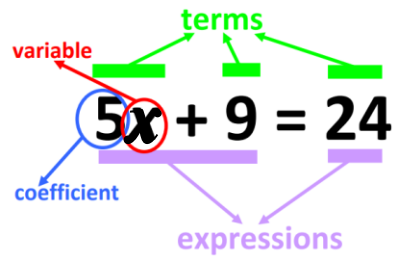
Constant – A number on its own

Coefficient – A number that is multiplied by a variable e.g. $8y$ -> 8 is the coefficient and y is the variable

Term – Either a single number, a variable, or number and/or variables multiplied together

Expression – A term or a combination of terms

Equation – A mathematical sentence starting that two expressions are equal



Rearranging Formulae

The **subject** of a formula is the variable that is being worked out. It can be recognised as the letter on its own on one side of the equals sign.

For example, in the formula for the area of a rectangle $A = bh$ (**area = base \times height**), the subject of the formula is A .

Rearrange the formula $v = u + at$ to make t the subject of the formula.

$$v = u + at$$

$$-u \quad -u$$

$$v - u = at$$

$$\div a \quad \div a$$

$$\frac{v - u}{a} = t$$

The letter t is now isolated, so t is now the subject of the formula.

Inequalities

$>$ Greater Than

$<$ Less Than

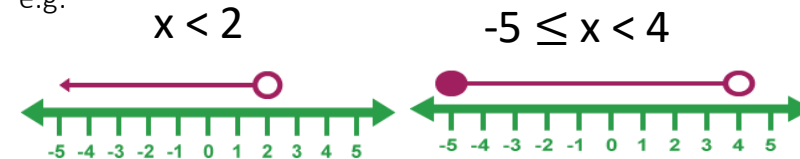
\geq Greater Than or Equal To

\leq Less Than or Equal To

Writing inequalities on a number line :

- Place dot on the numbers given in the inequality
- Colour in dot if your sign is a greater/less than or equal to
- Draw the line to satisfy the inequality

e.g.

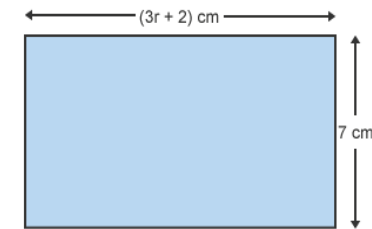


The process to solve inequalities is the same as the process to solve equations.

$$\begin{array}{r} 3m + 2 > -4 \\ -2 \quad -2 \\ \hline 3m > -6 \\ \div 3 \quad \div 3 \\ \hline m > -2 \end{array}$$

Forming and Solving Equations

The area of this rectangle is 56 cm^2 . Find the value of r .



Area of a rectangle = base \times height. This means $3r + 2$ will all be multiplied by 7. To show this in algebra, use a bracket for $3r + 2$ to show that both terms are being multiplied by 7.

7 multiplied by $(3r + 2)$ can be written as $7(3r + 2)$ as multiplication signs are not used in algebra.

$\text{Area} = \text{base} \times \text{height}$

$$\text{Area} = 7(3r + 2)$$

The area of the rectangle has been given in the question as 56 cm^2 :

$$56 = 7(3r + 2)$$

Expand the bracket:

$$56 = 7 \times 3r + 7 \times 2$$

$$56 = 21r + 14$$

Isolate $21r$ by subtracting 14 from both sides:

$$56 - 14 = 21r + 14 - 14$$

$$42 = 21r$$

Isolate r by dividing both sides by 21:

$$42 \div 21 = 21r \div 21$$

$$2 = r$$

Solving Linear Equations

Solve the equation $4y + 5 = -3$.

$$4y + 5 = -3$$

Subtract 5 from each side:

$$4y + 5 - 5 = -3 - 5$$

Simplify:

$$4y = -8$$

Get y by itself by dividing both sides by 4:

$$4y \div 4 = -8 \div 4$$

$$y = -2$$

Solve the equation $5(2c - 3) = 19$.

Expand the bracket:

$$5 \times 2c - 5 \times 3 = 19$$

$$10c - 15 = 19$$

Isolate $10c$ by adding 15 to each side:

$$10c - 15 + 15 = 19 + 15$$

$$10c = 34$$

Isolate c by dividing by 10:

$$10c \div 10 = 34 \div 10$$

$$c = \frac{34}{10} = \frac{17}{5} \text{ or } 3.4$$

GCSE Foundation Topic 4 Equations 1 Student Knowledge Organiser

Solving linear equations

Solve

(a) $2x + 3 = 9$ (b) $3w - 1 = 14$ (c) $7y + 2 = 30$

$3(2a + 1) = 21$ $5(4a - 3) = 65$

(a) $\frac{x+1}{2} = 9$ (b) $\frac{x-3}{4} = 8$ (c) $\frac{m-8}{5} = 3$

(a) $4x + 15 = x + 3$ (b) $8x + 40 = 3x + 5$ (c) $9x + 7 = 11x + 20$

(d) $7x + 9 = 2x - 16$ (e) $9x - 70 = 2x - 91$ (f) $4 - 5x = 3x + 28$

Rearranging formulae

Make x the subject

(a) $4x + c = w$ (b) $dx - t = 8$ (c) $x^2 + 3 = h$

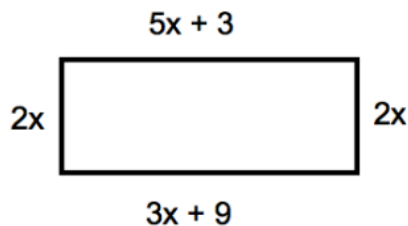
(d) $2x + 2y = p$ (e) $s = x^2 - 3$ (f) $y = xz + s$

(j) $3y = 4x + 1$ (k) $x^2 + a = v$ (l) $x^3 - 4 = 5y$

(m) $\frac{x+t}{m} = 2c$ (n) $\frac{w+x}{u} = 3z$ (o) $A = \pi x^2$

Forming and solving equations

Fiona is x years old. Thomas is 3 years older than Fiona. Cara is twice as old as Fiona. The sum of their ages is 51.
 (a) Form an equation in terms of x and solve to find their ages



- a) Form an equation and solve to find x
 b) Find the area of the rectangle

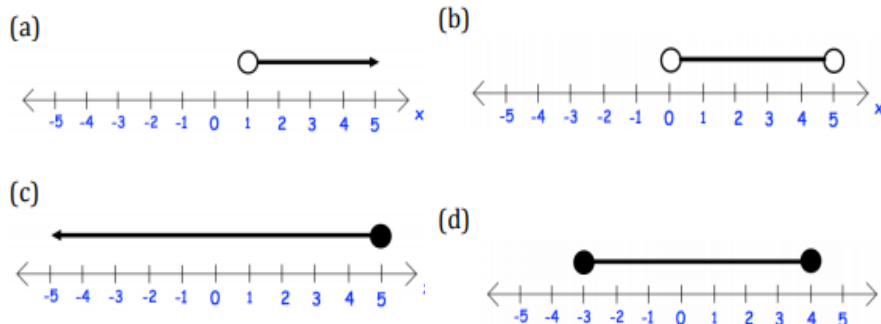
Inequalities

Solve

(a) $2x + 1 \leq 9$ (b) $3x - 5 > 16$ (c) $4x + 8 < 32$

(d) $4(x - 2) < 18$ (e) $2(2x - 9) \geq 22$ (f) $3(2x + 7) \leq 9$

Write down the inequality shown on the number line

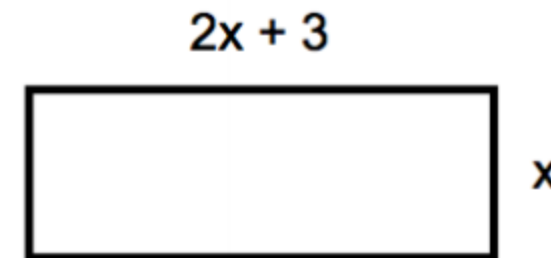


Applying Knowledge

The sum of each row is given.
 Find a , b , c and d .

a	a	a	a	24
a	a	b	b	28
b	c	c	c	29
a	b	c	d	31

Below is a rectangle, with width x cm and length $2x + 3$ cm. The perimeter of the rectangle is 72 cm. Calculate the area.



Write down all the integer values of x that satisfies $-2 \leq 2x < 6$

x is an integer.

Write down all the solutions of the inequality $3 < 2x + 1 < 13$

GCSE Foundation Topic 5 Charts and averages Student Knowledge Organiser

Key words and definitions

- Frequency – How many times a value occurs
- Cumulative Frequency – Frequency added together
- Ascending – Going up from smallest to biggest
- Median – Middle value in an ascending list of data
- Mode/Modal value – most common value in the data
- Mean - The total of the numbers divided by how many numbers there are.
- Range – Biggest number – smallest number
- Sum - addition of values

Averages from lists

7 babies weigh the following amounts:
2.5 kg, 3.1 kg, 3.4 kg, 3.5 kg, 3.5 kg, 4 kg, 4.1 kg

$$\bullet \text{ mean} = \frac{2.5+3.1+3.4+3.5+3.5+4+4.1}{7} = \frac{24.1}{7} = 3.44 \text{ (2 dp)}$$

• 2.5 kg, 3.1 kg, 3.4 kg, 3.5 kg, 3.5 kg, 4 kg, 4.1 kg
The median weight of these babies is 3.5 kg.

• 2.5 kg, 3.1 kg, 3.4 kg, 3.5 kg, 3.5 kg, 4 kg, 4.1 kg
The modal weight is 3.5 kg.

Averages from table

	Number of Goals	Frequency	Cumulative Frequency
	0	2	2
	1	3	5
	2	5	10
	3	1	11
Total		11	

Mode = category with biggest frequency = **2 goals**

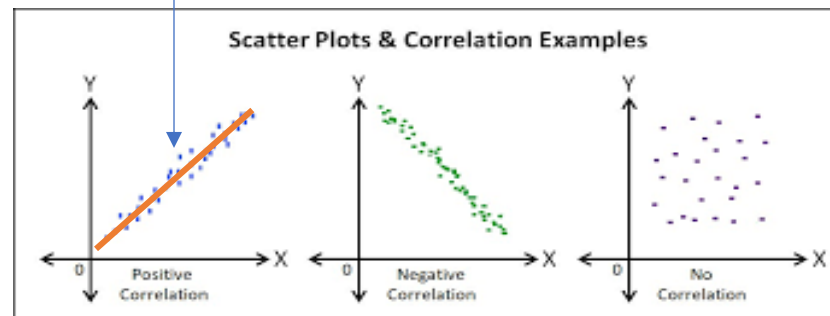
Median = value in the $\frac{\text{Total}+1}{2}$ position = 6th position = **2 goals**

$$\text{Mean} = \frac{\text{Sum of frequency} \times \text{number of goals}}{\text{Total}} = \frac{0 \times 2 + 1 \times 3 + 2 \times 5 + 3 \times 1}{11} = \frac{16}{11} = \mathbf{1.5 \text{ goals (1.d.p)}}$$

For grouped data, $0 \leq m < 4$ use the middle value when multiplying the data by the frequency when calculating the mean.

Scatter Graphs

Use a line of best fit to show correlation and to estimate values using the scatter graph



Reverse mean

The mean height jumped by a high jumper after 10 jumps is 1.81m. He jumps another jump at 1.73m, what is his new mean height?

$$1.81 \times 10 = 18.1\text{m} = \text{Sum of all 10 jumps}$$

$$\text{Mean of 11 jumps} = \frac{\text{Sum of 11 jumps}}{\text{Total no. of jumps}}$$

$$= \frac{18.1+1.73}{11} = 1.80\text{m (2.d.p)}$$

Pie Charts

To draw a pie chart, find the proportion of 360° :

$$1 \text{ item/frequency} = \frac{360^\circ}{\text{Total Frequency}} = \frac{360^\circ}{180} = 2^\circ$$

People travelling in a vehicle	Frequency	Calculation	Angle
1 person	120	2×120	240°
2 people	40	2×40	80°
3 people	13	2×13	24°
4 people	5	2×5	10°
5 or more people	2	2×2	4°
Total	180		



Averages from lists

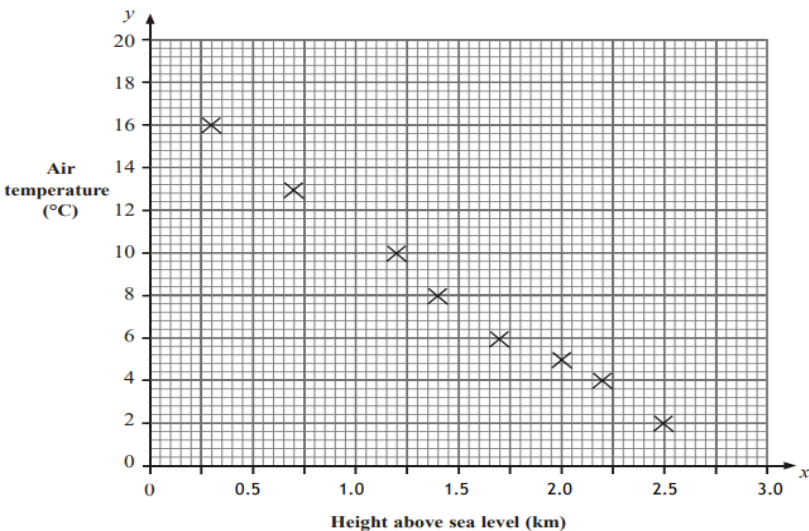
Here are 12 test scores of Jessica.

8 9 8 7 9 6 5 5 8 7 5 8

- Find the mean
- Find the median
- Find the mode
- Find the range

Scatter Graphs

Air temperature at different heights above sea level



- Draw a line of best fit
- State the type of correlation between the air temperature and height above sea level.
- Estimate the value of the air temperature at a height of 1.8m above sea level

Pie Charts

The table gives information about the numbers of fish in a lake.

Fish	Frequency
Perch	10
Bream	23
Carp	39

Draw an accurate pie chart to show this information.

Averages from tables

Number of drawing pins	Frequency
29	2
30	5
31	2
32	1

Time taken (m minutes)	Frequency
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9

For each table above, calculate

- Mean
- Median
- Mode

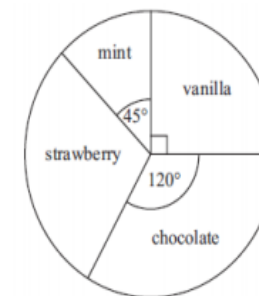
Reverse means

- Q1.** The mean number of goals scored by 3 players is 18. Another player joins, having scored 6 goals. What is the mean number of goals scored by the 4 players?
- Q2.** The mean of 9 numbers is 1.5. Another number is added. The mean is now 1.6. What number was added?

Applying Knowledge

- Q1.** Four numbers have a mean of 8 and a median of 8, but none of the numbers is 8. Give an example of what the four numbers could be.
- Q2.**

Some children were asked to name their favourite flavour of ice cream. The pie chart and table show some information about their answers.

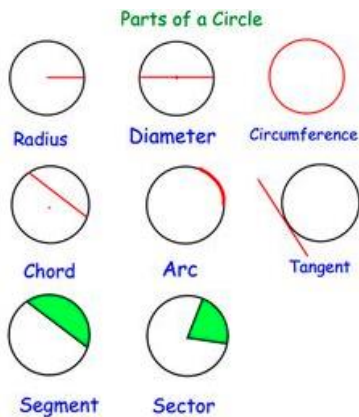


Use the pie chart to complete the table.

Flavour	Number of children	Angle of sector
vanilla	12	90°
mint	45°
strawberry	14
chocolate	120°

GCSE Foundation Topic 6 Area and volume 1 Student Knowledge Organiser

Key words and definitions



Volume

A measure of the amount of space occupied by an object.

Surface area

The area of all the faces in a 3D shape added together.

Compound shape

A shape made up of two or more basic shapes.

Prior Knowledge

Understand what is meant by area of a shape.

Understand what is meant by perimeter of a shape.

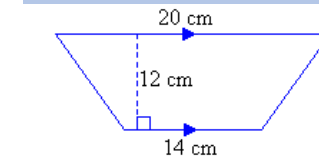
Calculate the area of a rectangle.

Calculate the area of a triangle.

Calculate the volume of a cuboid.

Calculate the volume of a prism.

Area



$a = 20 \text{ cm}$, $b = 14 \text{ cm}$, $h = 12 \text{ cm}$

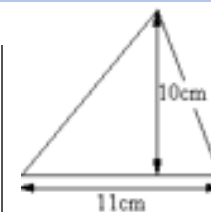
$$A = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(20+14) \times 12$$

$$= \frac{1}{2} \times 34 \times 12$$

$$= 204$$

So, the area of the trapezium is 204 cm^2

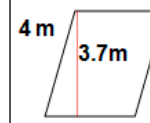


Area of triangle

$$= \frac{bh}{2}$$

$$= \frac{11 \times 10}{2}$$

$$= 55 \text{ cm}^2$$



Parallelogram

Area = bh

Area = $3.2 \text{ m} \times 3.7 \text{ m}$

Area = 11.84 m^2

Compound area

This figure can be separated into a rectangle and a semicircle. Find the area of each.

Rectangle: $A = L \times W$

$$A = 10 \times 14$$

$$A = 140 \text{ mm}^2$$

Semicircle: $A = \frac{\pi r^2}{2}$

$$A = \frac{3.14 \times 7^2}{2}$$

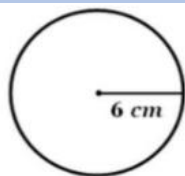
$$A = \frac{3.14 \times 49}{2}$$

$$A = 76.93 \text{ mm}^2$$

$$\text{Area} = 140 + 76.93$$

$$\text{Area} = 216.93 \text{ mm}^2$$

Circumference and area of a circle

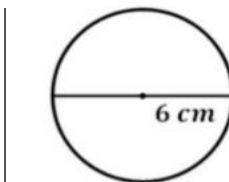


$$A = \pi r^2$$

$$= 3.142 \times 6^2 = \pi \times 6^2$$

$$= 3.142 \times 36 = 36 \pi$$

$$= 113.11 \text{ cm}^2$$



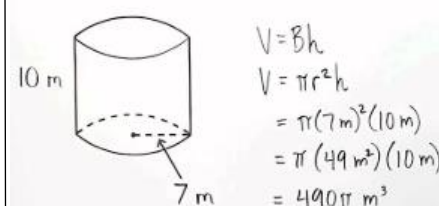
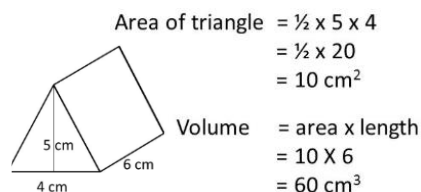
$$C = \pi d$$

$$= 3.142 \times 6 \text{ cm} = \pi \times 6$$

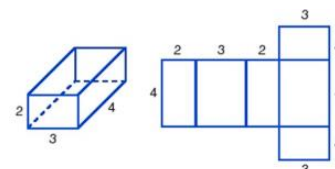
$$= 18.85 \text{ cm}$$

Volume of a prism

Volume of prism: $V = \text{area cross section} \times \text{length}$



Surface area of a prism

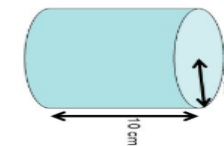


$$\text{S.A.} = 2(2 \times 4) + 2(3 \times 4) + 2(2 \times 3)$$

$$\text{S.A.} = 2(8) + 2(12) + 2(6)$$

$$\text{S.A.} = 16 + 24 + 12$$

$$\text{S.A.} = 52$$

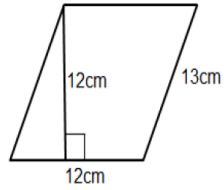
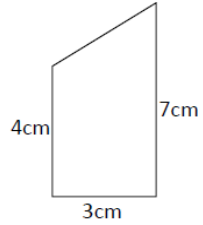
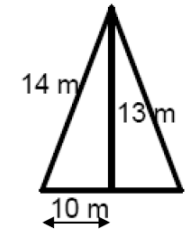


- Area of rectangle = $2\pi rh$
 $= 2 \times 3.14 \times 5 \times 10$
 $= 314 \text{ cm}^2$
- Area of two ends = $2\pi r^2$
 $= 2 \times 3.14 \times 5 \times 5$
 $= 157 \text{ cm}^2$
- Total surface area is $2\pi rh + 2\pi r^2$
- Total surface area = $314 + 157$
 $= 471 \text{ cm}^2$

GCSE Foundation Topic 6 Area and volume 1 Student Knowledge Organiser

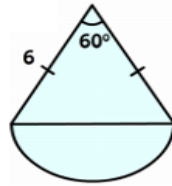
Area

Calculate area of the following shapes.



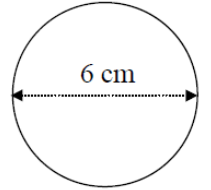
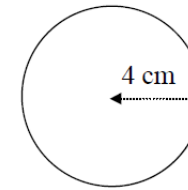
Compound area

Calculate area of the shape.



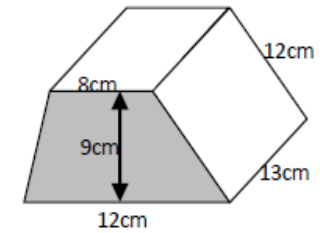
Circumference and area of a circle

Calculate the circumference and area of the circles.



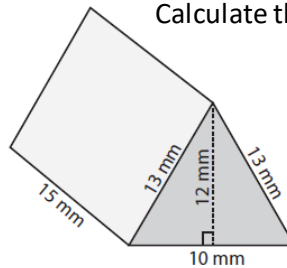
Volume of a prism

Calculate the volume of the trapezoid prism.



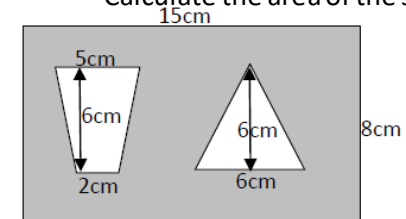
Surface area of a prism

Calculate the surface area of the triangular prism.



Applying knowledge

Calculate the area of the shaded area.



GCSE Topic 7 Fractions, decimals and percentages 1 Student Knowledge Organiser

Key words and definitions

Numerator - the top number of a fraction.

Denominator - the bottom number of a fraction, represents the number of parts to make one whole.

Equivalent - worth the same amount.

Simplify - reducing a fraction to an equivalent fraction with the lowest possible numerator and denominator.

Reciprocal - is one of a pair of numbers that when multiplied together gives the answer equal to 1.

Depreciation - the decrease in the value of something over time.

Interest - is money that is paid regularly at a particular percentage, usually when money has been lent or borrowed.

Multiply, divide, add and subtract fractions

$$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$$

Simplify first?

Multiply numerator

Multiply denominator

$$\frac{2}{15} \div \frac{4}{5} = \frac{2}{15} \times \frac{5}{4} = \frac{1}{6} = \frac{10}{60}$$

Simplify?

Flip the second fraction (reciprocal) and change to x

Multiply the fractions

Simplify?

$$\frac{2}{7} + \frac{3}{5} = \frac{10}{35} + \frac{21}{35} = \frac{31}{35}$$

Make common denominator

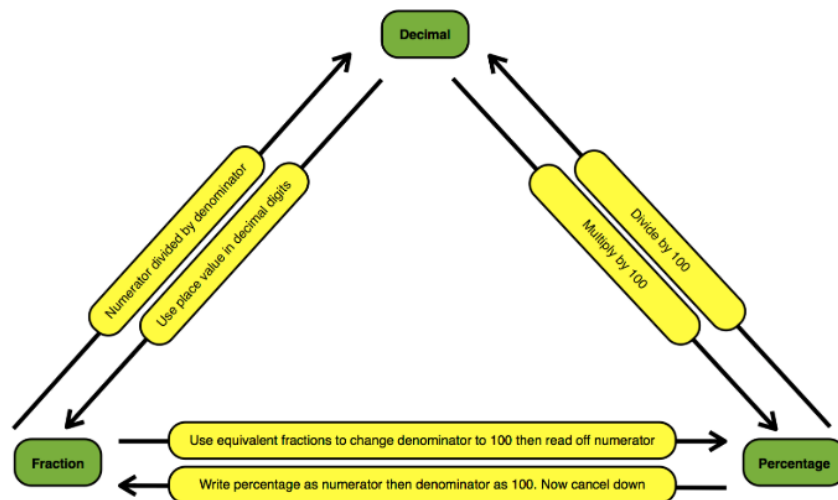
$$\frac{7}{9} - \frac{3}{4} = \frac{28}{36} - \frac{27}{36} = \frac{1}{36}$$

Do same to numerator as the denominator

Add/subtract numerators - keep the same denominator

Simplify?

Equivalent fractions, decimals and percentages



Percentage increase and decrease and percentage change.

Increase £240 by 15%

$$\begin{aligned} &100\% \text{ of } \pounds 240 \\ &+ 15\% \text{ of } \pounds 240 \\ \hline &115\% \text{ of } \pounds 240 \\ &= 240 \times 1.15 \\ &= \pounds 276 \end{aligned}$$

Or multiply by 1.15

Decrease £90 by 12%

$$\begin{aligned} &100\% \text{ of } \pounds 90 \\ &- 12\% \text{ of } \pounds 90 \\ \hline &88\% \text{ of } \pounds 90 \\ &= 90 \times 0.88 \\ &= \pounds 79.20 \end{aligned}$$

Or multiply by 0.88

$$\% \text{ change} = \frac{\text{change}}{\text{orig value}} \times 100$$

Equivalent Fractions

Fractions that have the same value, eg $\rightarrow \frac{2}{5} = \frac{20}{50}$

“What I do to the top, I do to the bottom”

Place the following in order $\frac{3}{8}$ $\frac{2}{5}$ $\frac{1}{4}$
Equivalent fractions with a common denominator are $\frac{15}{40}$ $\frac{16}{40}$ $\frac{10}{40}$

Answer $\frac{1}{4}$ $\frac{3}{8}$ $\frac{2}{5}$

Reverse Percentages

The cost of a television is £540 including a 20% sales tax. Work out the cost of the television without tax.

$$\begin{aligned} \text{original price} \times 1.2 &= 540 \\ \text{original price} &= 540 \div 1.2 \\ \text{original price} &= \pounds 450 \end{aligned}$$

The cost of a holiday is reduced by 15% to £833. What was the original price of the holiday?

$$\begin{aligned} \text{original price} \times 0.85 &= 833 \\ \text{original price} &= 833 \div 0.85 \\ \text{original price} &= \pounds 980 \end{aligned}$$

GCSE Foundation Topic 7 Fractions, decimals and percentages 1 Student Knowledge Organiser

Multiply, divide, add and subtract fractions

1. $\frac{3}{8} \times \frac{3}{4}$

6. $3\frac{9}{10} \div 2\frac{2}{3}$

2. $\frac{4}{5} \div \frac{2}{10}$

7. $6\frac{2}{5} + 2\frac{2}{3}$

3. $\frac{2}{9} + \frac{4}{6}$

8. $4\frac{5}{11} - 1\frac{1}{2}$

4. $\frac{3}{4} - \frac{3}{10}$

5. $4\frac{1}{4} \times \frac{1}{5}$

Equivalent fractions, decimals and percentages

In each of the following, four of the values are equal to each other. Which are they?

a) $\frac{7}{10}$ 0.375 $\frac{7}{20}$ 0.720 $\frac{3}{8}$ $37\frac{1}{2}\%$
 71% $\frac{17}{20}$ 38% 56% 0.3750 27%

b) $\frac{3}{5}$ 0.035 37% $\frac{3}{7}$ 73% 0.731
 65% 0.600 $\frac{1}{60}$ 60% 0.6 $\frac{1}{6}$

c) $\frac{2}{7}$ 0.071 0.27 $\frac{27}{100}$ 0.654 27%
 $\frac{54}{200}$ 54% 0.876 0.027 $\frac{6}{54}$ $2\frac{7}{10}\%$

d) 63% $\frac{3}{16}$ 0.603 $\frac{6}{13}$ 36% 0.72
 $\frac{9}{25}$ 0.925 0.036 0.36 $\frac{18}{50}$ 40%

Percentage increase and decrease and percentage change.

Increase £20 by 52%

Increase £32.10 by 17%

Increase 92kg by 110%

Decrease 21 kg by 7%

Decrease 110 lbs by 53%

Increase £110 by 7%, then reduce by 5%

Increase £400 by 6%, then by 6% again

- Find the percentage increase when:
 - a price of £10 is increased to £12.
 - a price of £20 is increased to £52.
- Find the percentage decrease when:
 - a price of £10 is decreased to £8.
 - a price of £25 is decreased to £22.

Equivalent Fractions

Find the missing numbers to make equivalent fractions

(a) $\frac{6}{7} = \frac{42}{\quad}$ (b) $\frac{9}{20} = \frac{63}{\quad}$ (c) $\frac{5}{12} = \frac{35}{\quad}$ (d) $\frac{7}{8} = \frac{\quad}{64}$

Arrange the fractions in order, smallest first.

(a) $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{3}$

(b) $\frac{1}{4}, \frac{3}{8}, \frac{1}{6}, \frac{5}{12}$

Reverse Percentages

- 20% of all the children in a class are left handed. 4 children are left handed. How many children are there in the class altogether?
- 30% of the members of a tennis club are pensioners. 36 members are pensioners.
 - How many members are there in total?
 - How many members are not pensioners?

Applying Knowledge

Shown below is a "magic square"
 Each column, row and diagonal has the same total.
 Work out the missing fractions.

$\frac{1}{10}$		$\frac{3}{10}$
$\frac{9}{20}$		
$\frac{1}{5}$	$\frac{3}{20}$	

Leonie bought a hat and a coat.
 The hat cost £6
 She sold both items for a total of £45
 Leonie made 300% profit on the hat and 125% profit on the total cost.
 Work out her percentage profit on the cost of the coat.

GCSE Foundation Topic 8 Ratio Student Knowledge Organiser

Key words and definitions

Ratio – ratio compares the size of one part to another part.

Proportion – compares the size of one part to the size of the whole.

Speed – the rate at which something moves.

Density – the mass of a substance per unit volume.

Pressure – the force per unit area exerted on an object.

Simplifying a Ratio

Ratios can be simplified, similar to fractions, by dividing each number in the ratio by their highest common factor (HCF).

Simplify the Ratio 6 : 15

Divide both our number values by **HCF (3)**

$$\begin{array}{ccc} 6 & : & 15 \\ \div 3 & & \div 3 \\ \hline 2 & : & 5 \end{array}$$

The simplified Ratio Answer is 2 : 5 ✓

Proportion Problems - Recipes

When solving recipe problems, find out how many ingredients are needed to make 1 of something, then multiply by how many you need.

Eg. To make 3 sponge cakes...

To make 2 sponge cakes		1 cake	3 cakes
½ pint	milk	¼ pint	¾ pint
2 lb	plain flour	1 lb	3 lb
4	eggs	2	6
20 ounces	sugar	10 oz	30 oz
20 ounces	butter	10 oz	30 oz

Operations: $\div 2$ and $\times 3$

Speed, Density and Pressure

Distance Speed Time

Speed = $\frac{\text{Distance}}{\text{Time}}$

Distance = Speed x Time

Time = $\frac{\text{Distance}}{\text{Speed}}$

Using each triangle, cover the measurement that you are trying to find. This will derive the given formulae.

Mass Density Volume

Volume = $\frac{\text{Mass}}{\text{Density}}$

Density = $\frac{\text{Mass}}{\text{Volume}}$

Mass = Density x Volume

Force Area Pressure

Pressure = $\frac{\text{Force}}{\text{Area}}$

Area = $\frac{\text{Force}}{\text{Pressure}}$

Force = Area x Pressure

Timetables

The table shows part of a bus timetable from Shotton to Alton.

Shotton	07 30	08 00	09 00	10 00	11 00
Crook	07 45	08 15	09 15	10 15	11 15
Prudhoe	07 58	08 28	09 28	10 28	11 28
Hexham	08 15	08 45	09 45	10 45	11 45
Alton	08 30	09 00	10 00	11 00	12 00

Serena lives in Crook. She has to be in Hexham by 11:15. What is the time of the latest bus she can catch from Crook to arrive in Hexham by quarter past 11?

The bus, which arrives in Hexham at 10:45, leaves Crook at 10:15.

Ratio Problems - Maps

When solving problems with map scales, label the ratio "map : real life" and scale up/down as needed.

Eg. If the scale is 1cm : 200m, what is the distance from the golf club to the cricket club?



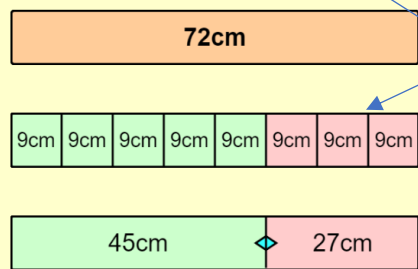
Map : real life

$$\begin{array}{ccc} \times 2 & \left(\begin{array}{c} 1\text{cm} : 200\text{m} \\ 2\text{cm} : 400\text{m} \end{array} \right) & \times 2 \end{array}$$

Sharing in a ratio

Share 72cm in the ratio 5:3.

Draw a bar model to calculate how much one part is worth.



$$5 + 3 = 8 \text{ parts}$$

$$72\text{cm} \div 8 = 9\text{cm per part}$$

$$5 \text{ parts} \times 9\text{cm} = 45\text{cm}$$

$$3 \text{ parts} \times 9\text{cm} = 27\text{cm}$$

Simplifying a Ratio

Write out and simplify the following ratios:

For every 6 women,
the school employs 8 men.

women : men

..... :



red squares : green circles

15cm to 75cm

400m to 1.5km

Ellie is making a cake.
The instructions say that the ratio of sugar to flour should be 1 : 3
Ellie uses 250g of sugar and 650g of flour.
Has Ellie used the correct ratio of sugar to flour?



Sharing in a Ratio

Share £60 in the
ratio 5:1.

Divide £48 in the
ratio 5:3.

Share £72 in the
ratio 4:5.

Divide £40 in the
ratio 3:5.

Share £132 in the
ratio 8:3.

The angles in a triangle are
in the ratio 1:5:6. Work out the
angles in degrees.

The ratio of boys to girls in
a class is 3:5.

Explain why there could not
be 30 pupils in the class.

William has a collection of coins. Each of
the coins is either silver or bronze.

The ratio of the number of bronze coins to
the number of silver coins is 4 : 1.

William has 12 **more** bronze coins than
silver coins. Work out the total number of
coins in his collection.

Over the course of a season, a football
team won, drew and lost matches in the ratio
2 : 1 : 5.

The team lost 12 **more** matches than they
won.

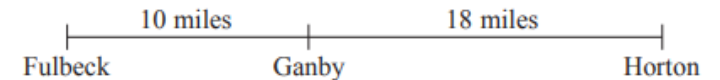
Work out how many matches the team drew
in the season.

Speed, Density and Pressure

A bus travels 222 miles in 6 hours.
What was the average speed of the bus?

Mr Jenkins catches the 11:45am bus from London to Glasgow.
The distance between the two cities is 407 miles.
The bus travels at an average speed of 55mph.
What time should he arrive in Glasgow?

The distance from Fulbeck to Ganby is 10 miles.
The distance from Ganby to Horton is 18 miles.



Raksha is going to drive from Fulbeck to Ganby.
Then she will drive from Ganby to Horton.
Raksha leaves Fulbeck at 10 00
She drives from Fulbeck to Ganby at an average speed of 40mph.
Raksha wants to get to Horton at 10 35
Work out the average speed Raksha must drive at from Ganby to Horton.

A cube of ice has side length of 5cm.
The mass of the cube of ice is 114.5g.

Find the density of ice.
Give your answer in g/cm^3

A box is placed on the floor.

The area of the box in contact with the floor is 2.4m^2
Pressure exerted on the floor 16 newtons/ m^2

Work out the force exerted by the box on the floor.

GCSE Foundation Topic 9 Shapes and angles Student Knowledge Organiser

Key words and definitions

Polygon – a plane figure with at least three straight sides and angles, and typically five or more.

Quadrilateral – 4 sided shape.

Pentagon – 5 sided shape.

Hexagon - 6 sided shape.

Heptagon – 7 sided shape.

Octagon – 8 sided shape.

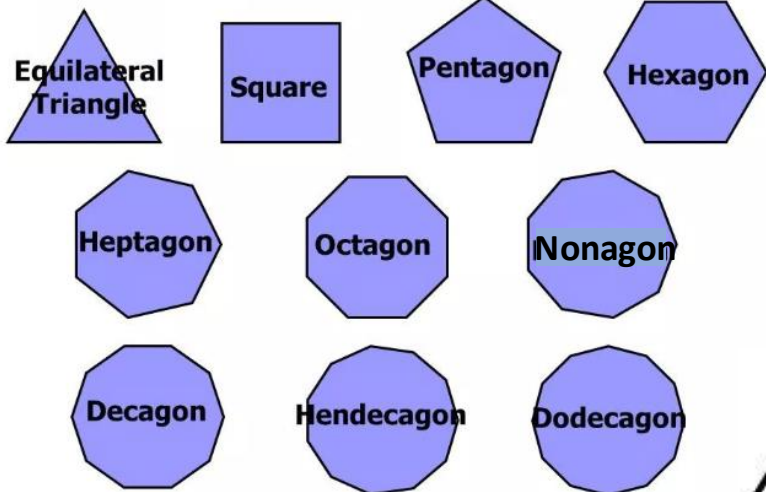
Nonagon – 9 sided shape.

Decagon - 10 sided shape.

Hendecagon – 11 sided shape.

Dodecagon – 12 sided shape.

Polygons

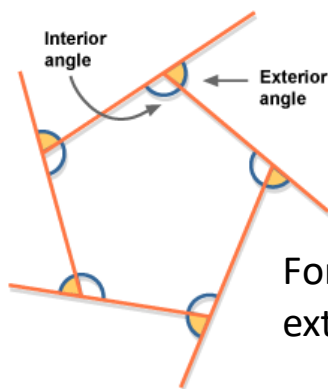


Prior Knowledge

Angles on straight lines/internal angle sums in polygons
Angles in parallel lines

Interior and exterior angles of polygons

Sum of interior angles = $180^\circ \times (n - 2)$
 $n = \text{number of sides}$

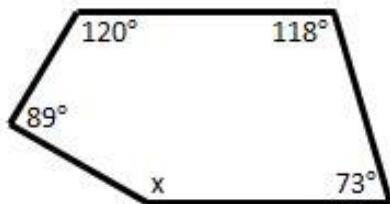


For all polygons the exterior angles total 360°

A regular polygon has an exterior angle of 20° .

How many sides does it have?

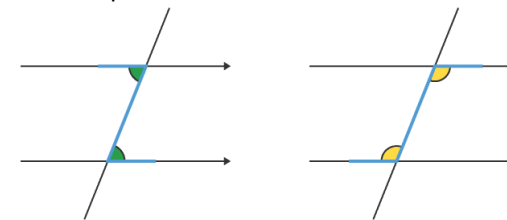
$$\begin{aligned} \text{Number of sides} &= 360^\circ \div 20^\circ \\ &= 18 \text{ sides} \end{aligned}$$



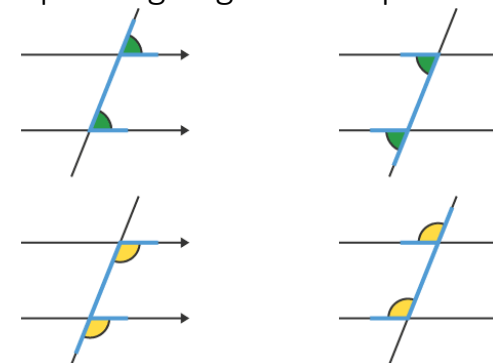
$$\begin{aligned} \text{Sum of angles} &= 89^\circ + 120^\circ + 118^\circ + 73^\circ \\ &= 400^\circ \\ \text{Sum of interior angles} &= 180^\circ \times (5 - 2) \\ &= 540^\circ \\ x &= 540^\circ - 400^\circ \\ &= 140^\circ \end{aligned}$$

Angles in parallel lines

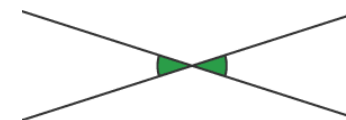
Alternate angles are equal



Corresponding angles are equal



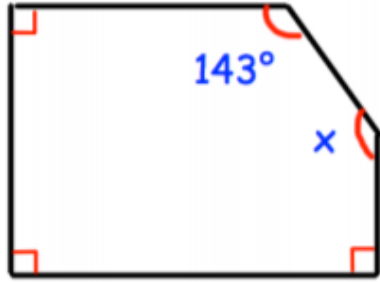
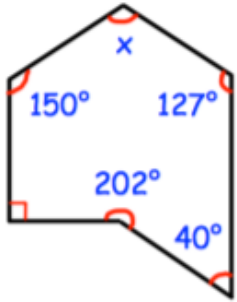
Vertically opposite angles are equal



GCSE Foundation Topic 9 Shapes and angles Student Knowledge Organiser

Interior and exterior angles of polygons

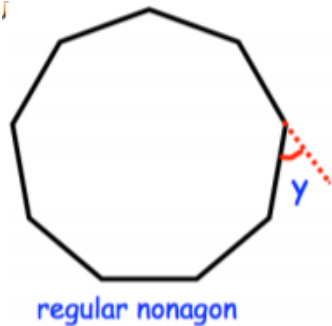
Find the missing angle in each irregular polygon



Work out the number of sides of polygons with these sum of interior angles

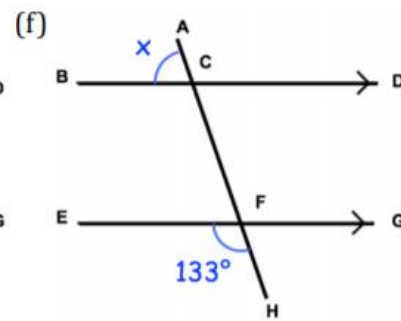
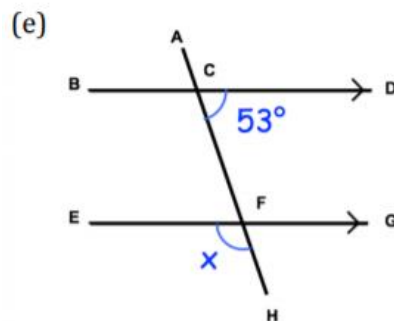
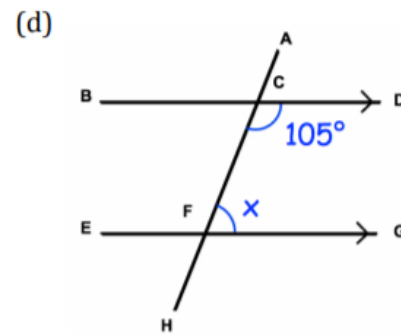
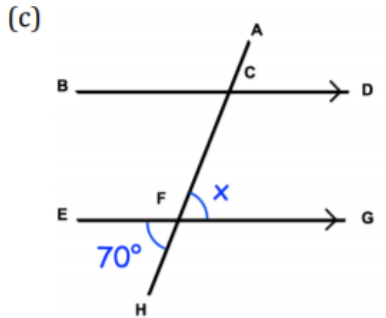
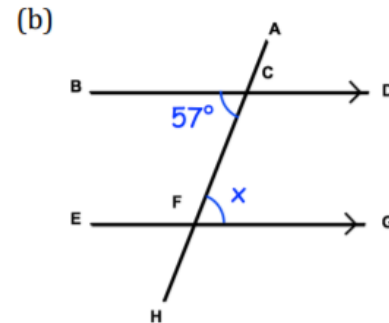
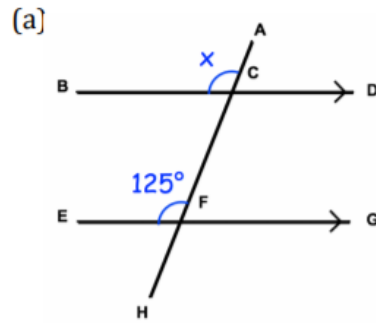
- (a) 1260° (b) 2880° (c) 3960°

Each of the polygons below are regular. Calculate the size of each exterior angle, y .



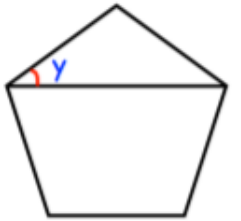
Angles in parallel lines

Find the angle x in each question below. Give reasons for your answer.



Applying knowledge

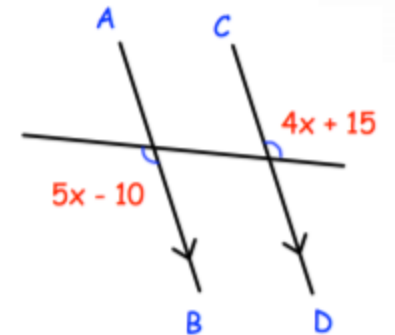
Shown is a regular pentagon. Find y .



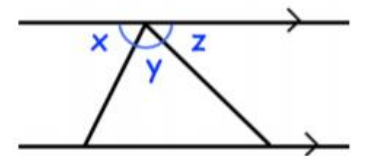
Explain why this cannot be an interior angle from regular polygons.



Find x



Matilda is proving that the angles in a triangle add up to 180° . She has started with this diagram. Complete her proof.



Key

Formula

$$a^2 + b^2 = c^2$$

a = side of right triangle

b = side of right triangle

c = hypotenuse

The **hypotenuse** (h) is the longest side. It is opposite the right angle.

The **opposite side** (o) is opposite the angle in question (x).

The **adjacent side** (a) is next to the angle in question (x).

Trigonometric Formula

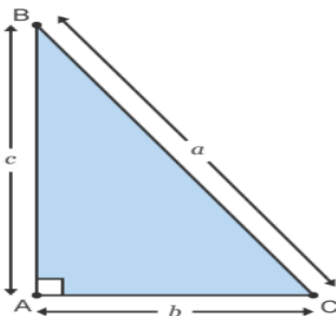
What are the formulas for sin cos and tan?

- $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan x = \frac{\text{opposite}}{\text{adjacent}}$

Pythagoras

Right-angled triangles

Pythagoras' theorem states that for all right-angled triangles, **'The square on the hypotenuse is equal to the sum of the squares on the other two sides'**. The hypotenuse is the longest side and it's always opposite the right angle.

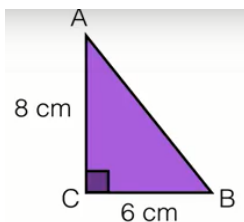


In this triangle $a^2 = b^2 + c^2$ and angle A is a right angle.

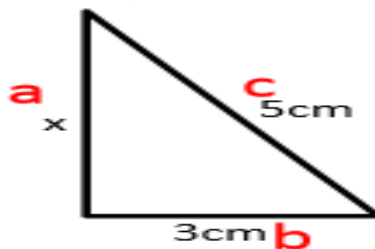
Pythagoras' theorem only works for right-angled triangles, so you can use it to test whether a triangle has a right angle or not.

In the triangle above, if $a^2 < b^2 + c^2$ the angle A is acute.

In the triangle above, if $a^2 > b^2 + c^2$ the angle A is obtuse.

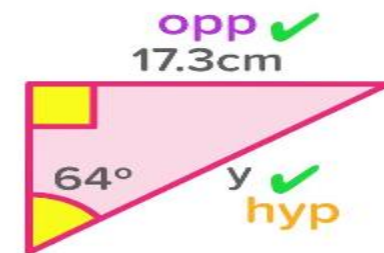
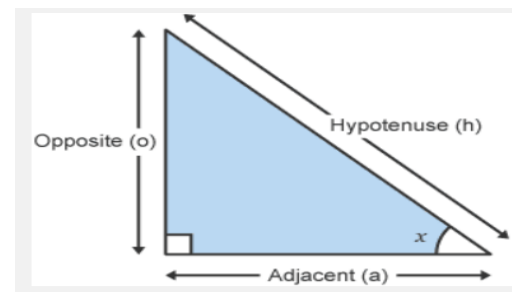


$$\begin{aligned} AB^2 &= BC^2 + AC^2 \\ AB^2 &= 6^2 + 8^2 \\ AB^2 &= 36 + 64 \\ AB^2 &= 100 \\ AB &= \sqrt{100} \\ AB &= 10 \text{ cm} \end{aligned}$$

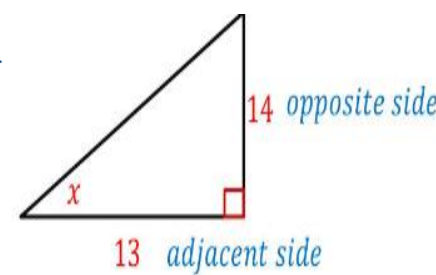


$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + 3^2 &= 5^2 \\ x^2 + 9 &= 25 \\ x^2 &= 25 - 9 \\ x^2 &= 16 \\ x &= \sqrt{16} \\ x &= 4 \text{ cm} \end{aligned}$$

Trigonometry



$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} \\ \sin(64) &= \frac{17.3}{y} \\ y &= \frac{17.3}{\sin(64)} \\ y &= 19.24801... \end{aligned}$$

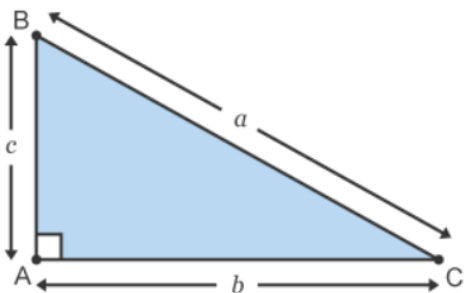


Use tangent ratio $\tan x = \frac{O}{A}$

Use inverse tangent $x = \tan^{-1}\left(\frac{14}{13}\right)$

Solve for x using calculator $x = 47.1^\circ$

Pythagoras

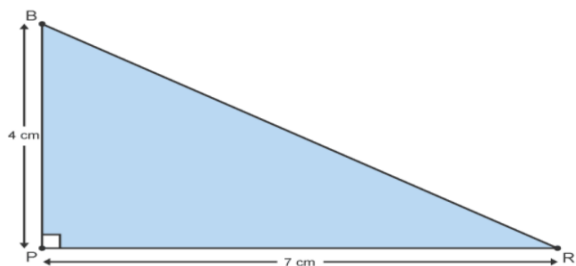


$$a^2 = b^2 + c^2$$

$$b^2 = a^2 - c^2$$

$$c^2 = a^2 - b^2$$

Work out the length of the line BR , correct to 1 decimal place.



A fireman has a ladder that is 13 metres long. If he wants to reach a window that is 12 metres above the ground, how far from the wall should he put the bottom of his ladder?

Peter's house is exactly 481m from school. To get home he walks 480m south and then he walks west. How far west does he have to walk?

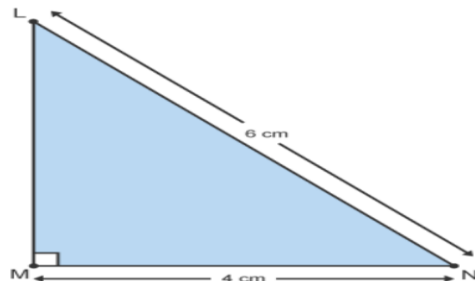
A triangle has sides of length 23.8cm, 31.2cm and 39.6cm.

Is this a right-angled triangle?

Show how you decide.

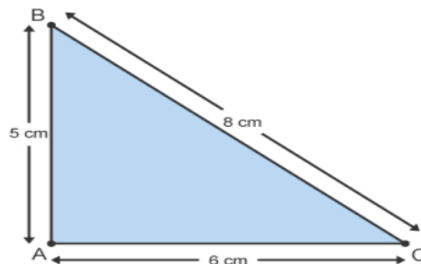
Pythagoras

Work out the length of the line LM , correct to 1 decimal place.



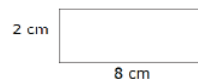
Which of the following triangles is right-angled?

a)

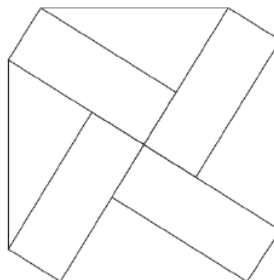


b)

Here is a rectangle.



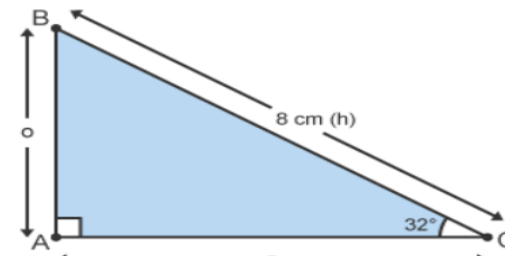
The 8-sided shape below is made from 4 of these rectangles and 4 congruent right-angled triangles.



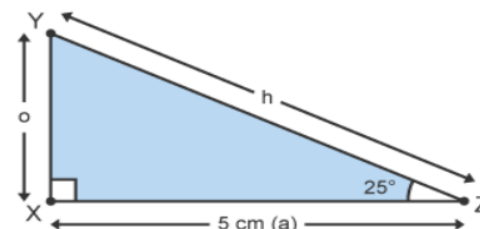
Work out the perimeter of the 8-sided shape.

Trigonometry

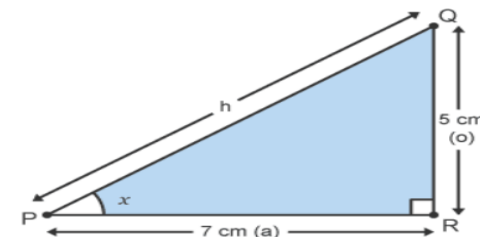
Calculate the length AB . Give the answer to one decimal place.



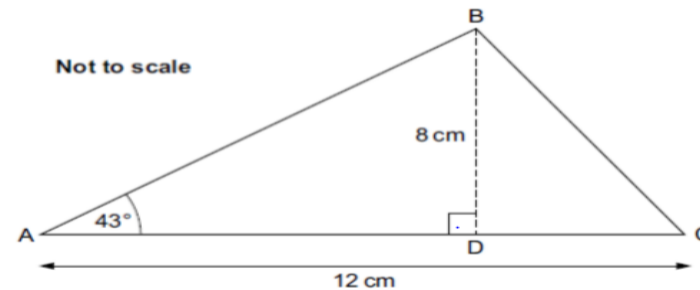
Calculate the length YZ . Give the answer to one decimal place.



Calculate the angle QPR . Give the answer to one decimal place.



Not to scale



Calculate angle BCA .

GCSE Foundation Topic 11 Sequences Student Knowledge Organiser

Key words and definitions

Difference – amount between two numbers,
e.g. difference between 8 and 6 = $8 - 6 = 2$

Term – A number in a sequence

Nth Term – The term at the nth position

Coefficient – A number in front of an algebraic term
e.g. coefficient of $3a^2$ is 3

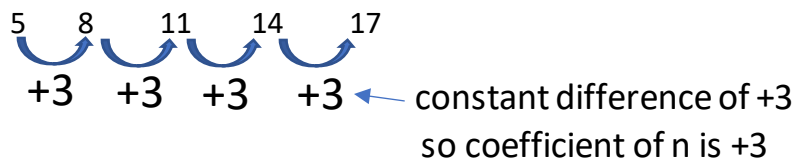
Linear sequence – has a constant difference between each term

Quadratic sequence – has a constant second difference between each term

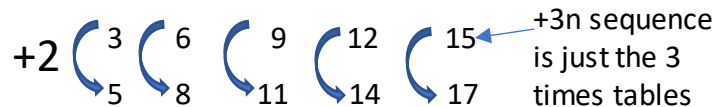
Geometric sequence – has a constant multiplier to get to the next term

Nth term of linear sequences

Find the nth term of the linear sequence below :



Compare with sequence of $+3n$

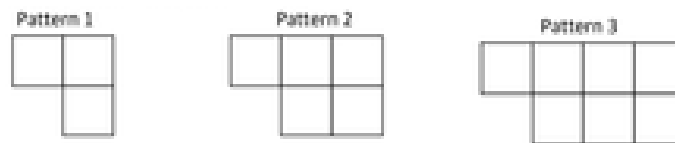


Nth term = $3n + 2$

For sequences that go up, you will have a **positive** coefficient of n.

For sequences that go down, you will have a **negative** coefficient of n.

Pattern recognition and continuation



When given patterns, turn them into number sequences to help continue and find the nth term of them.

So the sequence above becomes :

3 5 7 ...

Next pattern will include 9 squares.

Common Sequences to know

Square Numbers – 1 4 9 16 25 36 ...

Cube Numbers – 1 8 27 64 125 ...

Triangle Numbers – 1 3 6 10 15 ...

These numbers can be represented as a triangle of dots

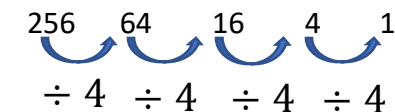
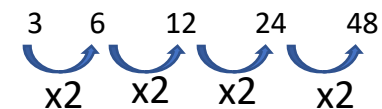
Fibonacci sequence - 1 1 2 3 5 8 ...

These numbers are created by adding the two previous

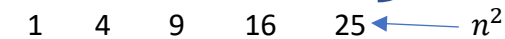
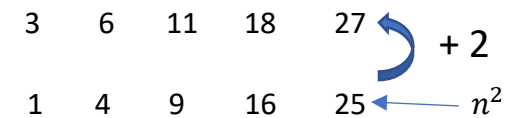
terms in the sequence to get the next term

Recognising other sequences

Geometric sequences either multiply or divide by the same number to get to their next term



Quadratic sequences are sequences that include an n^2 in the nth term. Use the square numbers sequence to help describe the rule.



So the rule is $n^2 + 2$ or *square numbers increased by 2*

GCSE Foundation Topic 11 Sequences Student Knowledge Organiser

Nth term of linear sequences

Find the nth term of these linear sequences :

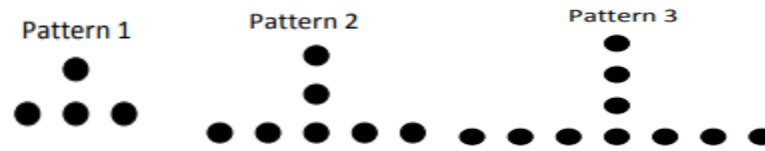
- a) 4 9 14 19 24
- b) 1 7 13 19 25
- c) 2 4 6 8 10
- d) 20 17 14 11 8
- e) 11 6 1 -4 -9
- f) -20 -24 -28 -32 -36

Other Sequences

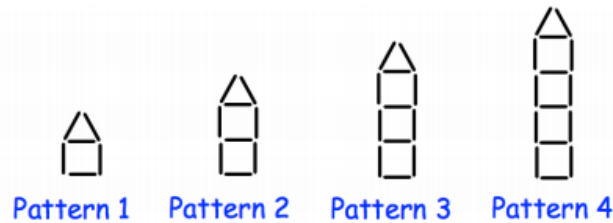
Find the next three terms of these sequences:

- a) 1 2 4 8 16
- b) 1 4 9 16 25
- c) 5 10 20 40 80
- d) 2 8 18 32 50
- e) $\frac{1}{10}$ $\frac{2}{9}$ $\frac{3}{8}$ $\frac{4}{7}$ $\frac{5}{6}$
- f) $\frac{1}{3}$ $\frac{1}{12}$ $\frac{1}{48}$ $\frac{1}{196}$

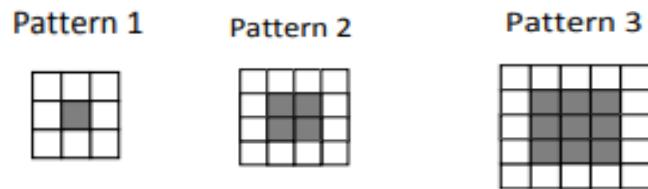
Pattern recognition and continuation



- a) How many dots are in pattern 4
- b) Find the nth term of the sequence of dots
- c) How many dots will be in pattern 15



- a) How many sticks will be used in pattern 5?
- b) Find the nth term for the number of sticks



- a) How many grey and white squares are in pattern 4
- b) Find the nth term for the number of white squares

Fibonacci Sequence

Find the next two terms of these Fibonacci style sequences.

- a) 2 4 6 10
- b) 5 12 17 29
- c) 3 7 10 17
- d) -1 -3 -4 -7

Applying knowledge

Q1. Find the 5th term in each of the linear sequences described below:

- a) $2n - 4$
- b) $3n + 6$
- c) $20 - 4n$

Q2. Jim claims that the term 387 is not in the sequence $4n - 1$. Explain why he is correct.

Q3. The first three terms of a different Fibonacci sequence are a b a + b

- a) Show that the 6th term of this sequence is $3a + 5b$
- b) Given that the 3rd term is 7 and the 6th term is 29, find the value of a and the value of b.

GCSE Foundation Topic 12 Probability Student Knowledge Organiser

Key words and definitions

Probability – the chance that a particular outcome will occur

Event – a single result of an experiment

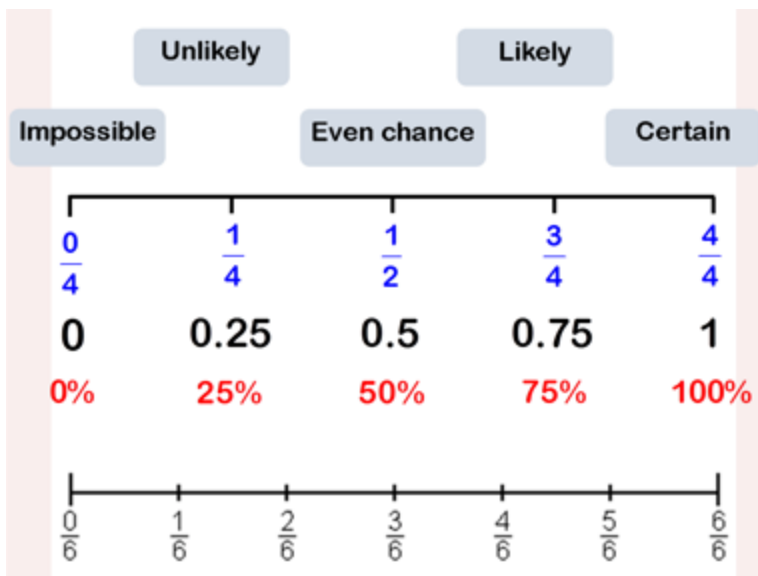
Outcome – one of the possible results of an experiment

Theoretical probability – the probability that an outcome will occur based on all possible outcomes

Experimental probability – derived from the results of an experiment. The total number of successes divided by the total number of trials

Sample space – all the outcomes of an event, presented in table form

Probability scales



Prior knowledge

Convert between fractions, decimals and percentages

Represent information in a table

Simple Probability

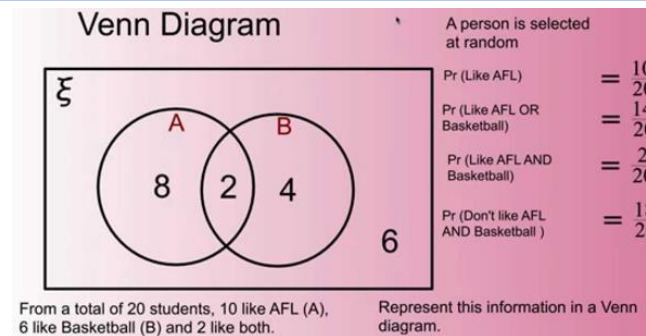
- The probability of an event, denoted $P(E)$, is the likelihood of that event occurring.

The Probability of an Event =

$$P(\text{Event}) = \frac{\text{the number of ways it can happen}}{\text{the number of possible outcomes}}$$

Example – when rolling a die,
 $P(4) = 1/6$ as there is 1 4, and 6 numbers on the die

Venn diagram



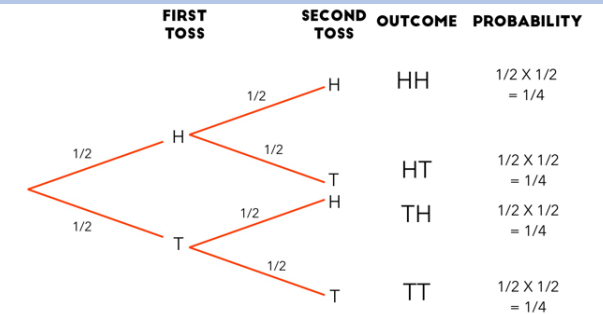
Sample space

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability of getting a total of ten = $\frac{3}{36}$

When rolling 2 dice and adding the scores, the sample space diagram looks like this. The probability of an event can be calculate by counting the number of favourable outcomes and dividing by the total number of outcomes

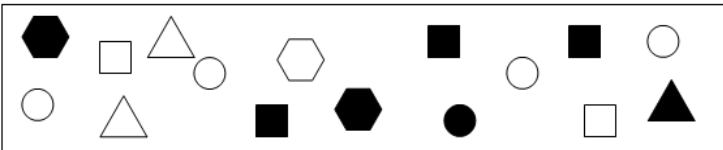
Tree diagrams



Multiply as you move along the branches

Basic probability

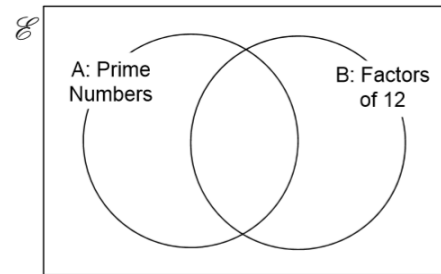
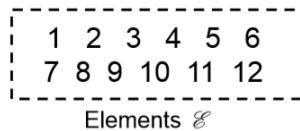
- The probability scale goes from 0 to 1. Write down what is meant by each of the following probabilities
 - 0 Answer: _____
 - $\frac{1}{2}$ Answer: _____
 - 1 Answer: _____
- A bag contains 4 Red, 3 Blue, 2 Green and 1 Yellow marbles. You ask a friend to pick out one marble at random. Calculate the following:
 - $P(\text{Red}) =$
 - $P(\text{Blue}) =$
 - $P(\text{Green}) =$
 - $P(\text{Yellow}) =$
 - $P(\text{Red or Green}) =$
 - $P(\text{Not Green}) =$
 - $P(\text{Black}) =$
- A box contains the shapes shown below. You ask a friend to pick out one shape at random. Calculate the following:



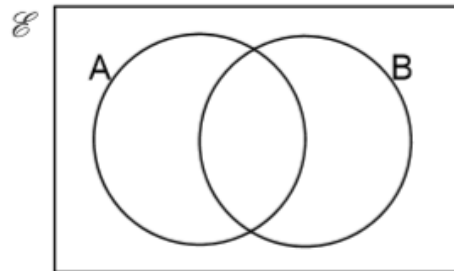
- $P(\text{Black Square}) =$
- $P(\text{Square}) =$
- $P(\text{Circle}) =$
- $P(\text{White Hexagon}) =$
- $P(\text{Black}) =$

Venn diagrams

① Place each element in the correct section of the Venn diagram.

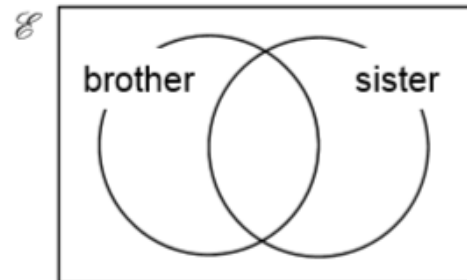


2. $E = \{8, 9, 10, 11, 12, 13, 14, 15, 16\}$
 $A = \{\text{even numbers}\}$
 $B = \{\text{square numbers}\}$
 (a) Complete the Venn diagram.



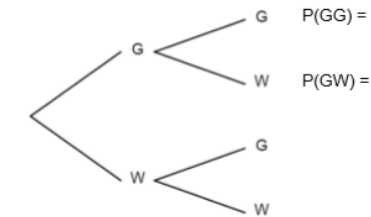
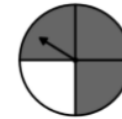
4. In a class of 32 pupils,
 23 pupils have a brother,
 14 pupils have a brother and a sister,
 6 pupils have no brothers or sisters.

Write the number of pupils who belong in each section of the Venn diagram.



Tree diagrams

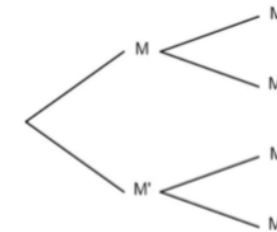
1. A spinner has four equally sized sectors: three grey and one white. The spinner is to be spun twice.
 (a) Complete the tree diagram.



- (b) Work out the probability that the spinner will land on the same colour on both spins.

- (c) Work out the probability that the spinner will land on grey on at least one of the spins.

3. Angela is playing a game with two fair dice. She rolls both dice and wins a point for each die that lands on a multiple of 3.
 (a) Complete the tree diagram, in which M stands for a multiple of 3.



- (b) Work out the probability that Angela scores 2 points.

- (c) Work out the probability that Angela scores at least 1 point.

- (d) Work out the probability that Angela scores no points.

Key words and definitions

Centre of Enlargement- The point that a shape is enlarged from.
 Centre of Rotation- The point that you place your pencil on the rotate a shape.

Enlargement- Making a shape bigger or smaller.

Rotation- Turning a shape around.

Reflection- Drawing the mirror image of a shape.

Scale Factor- How many times bigger or smaller you make a shape.

Transformation- Changing a shape.

Translation- Moving a shape.

Vector- A pair of numbers written one on top of the other that describe how a shape is translated.

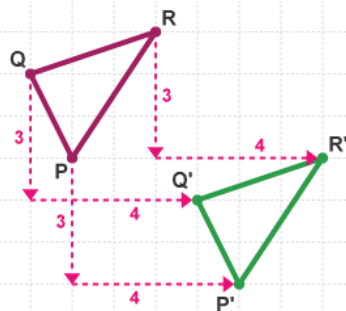


Clockwise



Anti-clockwise

Translations



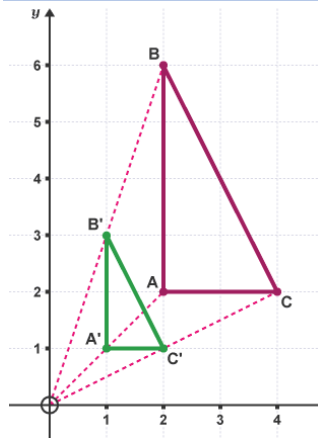
To translate a shape you move it a given number of squares, up down left or right.

These could be given as a vector, the first number in the vector saying how far left and right, and the second number saying how far up or down. Up and right as positive numbers, down or left are negative numbers.

E.g. In the diagram PQR has been translated to P'Q'R' by the vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

When describing a translation state the vector it has been translated by.

Enlargements



When enlarging a shape, you'll be given a **scale factor**, this is number that tells you how many times bigger a shape is getting. You will also be given a **centre of enlargement**, when you enlarge your shape the distance from the centre of enlargement is also enlarged.

If you get a fractional scale factor, it make the shape smaller not larger.

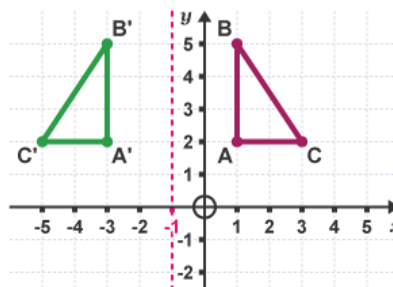
For example on the question above the shape ABC has been enlarged by $\frac{1}{2}$ and has a centre of enlargement at (0,0). If you've enlarged your shape correctly you should be able to draw a straight line from each corresponding corner to the centre of enlargement.

When describing an enlargement always state the scale factor and centre of enlargement, this can be found by drawing lines connecting the corresponding corners and seeing where they cross.

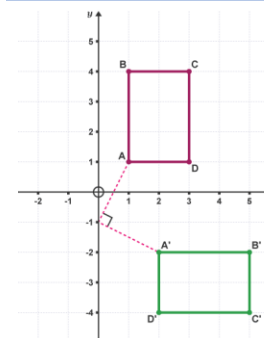
Reflection

To reflect a shape draw its mirror image on the opposite side of the reflection line, ensuring it's the same distance from the line as the original shape.

When describing a reflection always state the equation of the line it has been reflected.



Rotations

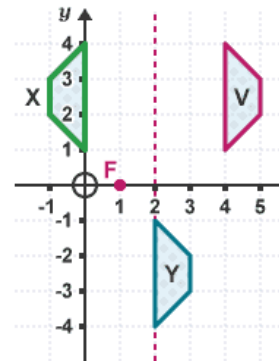


To rotate a shape:

- 1) Draw your shape onto tracing paper
- 2) Use your pencil to pin your tracing paper to the page with the tip on the centre of rotation.
- 3) Rotate the shape round the correct number of degrees. Then, using this as reference, draw the shape in the correct position.

When describing a rotation always state how many degrees its rotated, whether its clockwise or anti-clockwise, and what the centre of rotation is.

Combined



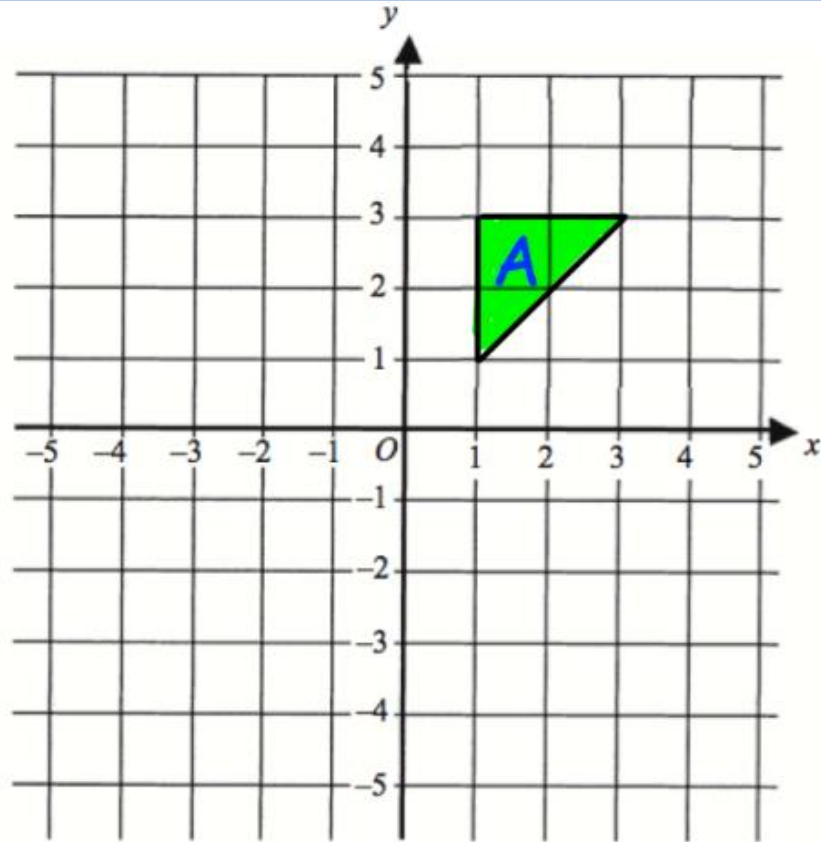
Sometimes transformations can be combined.

For example in this question V has been reflected in the line $x=2$ to get X, then X has been rotated 180° around the point (1,0).

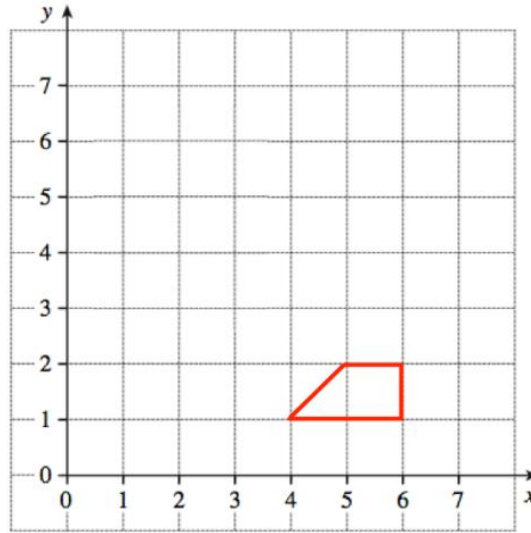
When doing multiple transformations do one part at a time, drawing each shape a long the way.

GCSE Foundation Topic 13 Transformations Student Knowledge Organiser

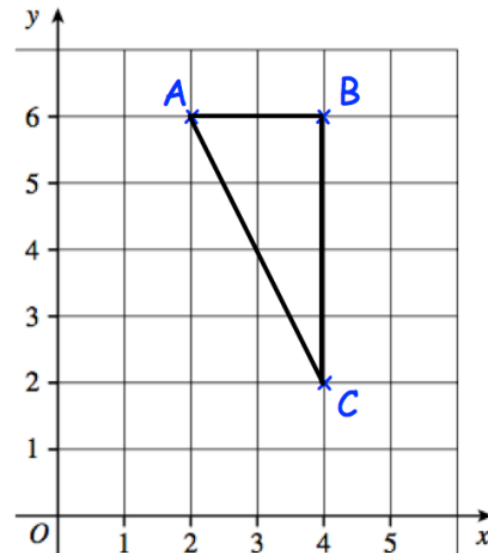
- a) Translate shape A by the vector $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$, name it shape B
- b) Reflect shape A in the line $y=-1$, name it shape C
- c) Rotate shape A 180° clockwise around $(0,0)$, name it shape D



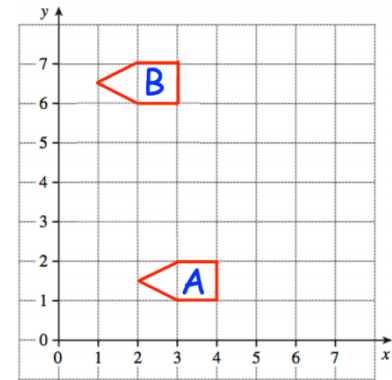
Enlarge the trapezium by a scale factor of 2, centre $(7,0)$



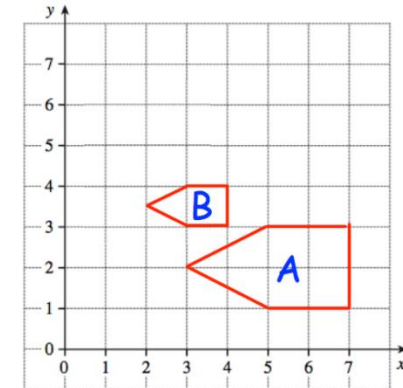
Enlarge the triangle by a scale factor of $\frac{1}{2}$, centre $(0,0)$



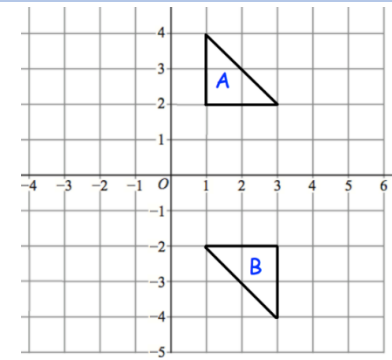
Describe the transformation of Shape A to Shape B



Describe the transformation of Shape A to Shape B



Describe the transformation of Shape A to Shape B



Key words and definitions

Expression - Mathematical expression that consists of variables, numbers and operations. Eg $4a + 2y$

Term - A term is either a single number or variable. Terms are separated by a + or - sign in an overall expression.

Simplify - condense an algebraic expression by grouping and combining similar terms.

Expand - Removing the (), the terms inside the bracket will be multiplied by the term outside of the bracket.

Factorise - Taking an algebraic expression and putting it back into brackets.

Expanding double brackets

F O I L

first outer inner last

multiply

$$(x + 8)(x + 5)$$

$$x^2 + 5x + 8x + 40$$

$$x^2 + 13x + 40$$

multiply

$$(2y - 6)(y + 7)$$

$$2y^2 + 14y - 6y - 42$$

$$2y^2 + 8y - 42$$

Factorising quadratic expressions

$$x^2 - 3x + 2$$

$$(x - 1)(x - 2)$$

$$x^2 + 3x - 10$$

$$(x + 5)(x - 2)$$

$$x^2 - x - 30$$

$$(x + 5)(x - 6)$$

Identify the factors of the last number, then combine to make the required amount of x. Be careful with the sign!

Expanding Single brackets

Expand the brackets

$$2(x + 8)$$

$$2x + 16$$

$$6y(9 - y)$$

$$54y - 6y^2$$

Expand the brackets and simplify

$$12(4x + 8) - 6x$$

$$48x + 96 - 6x$$

$$42x + 96$$

$$7(a - 11) + 2(3 + a)$$

$$7a - 77 + 6 + 2a$$

$$9a - 71$$

Factorising linear expressions

$$5x + 25$$

$$5(x + 5) \checkmark$$

$$5x + 25$$

Find the HCF of 5 and 25. This is put outside of the brackets. Work backward to decide the terms inside of the bracket eg. $5x$? Will give $5x$?

Difference of 2 squares

Factorise

$$a^2 - 81$$

$$= (a - 9)(a + 9)$$

One - and one +

Solving quadratic equations by factorising

Solve $(x + 7)(x - 2) = 0$

$$x + 7 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x + 7 - 7 = 0 - 7 \quad \Bigg| \quad x - 2 + 2 = 0 + 2$$

$$x = -7 \quad \Bigg| \quad x = 2$$

Expand

1. $5(x + 2)$
2. $4(x - 4)$
3. $6(2x + 1)$
4. $x(3x + 2)$
5. $3x(x - 10)$
6. $4(2 + x)$
7. $x(2y + 1)$
8. $10x(2x + 4)$
9. $4(6 - 3x)$
10. $12y(3x + 2)$
11. $(x + 2)(x + 5)$
12. $(x + 5)(x + 9)$
13. $(x + 4)(x + 1)$
14. $(x + 2)(x - 3)$
15. $(x - 3)(x + 5)$
16. $(x - 2)(x + 7)$
17. $(x - 3)(x - 5)$
18. Solve $x^2 + 10x + 21 = 0$

Factorise

Factorise
$7x + 14$
$45 - 27k$
$12ab + 7b$
$y^2 - 9y$
$8t - 32t^2$
$16gh + 28gf$
$21w^2z - 77wx$

Harder factorisation

Factorise $2y^2 + 7y - 15$

Factorise $3y^2 + 10y - 8$

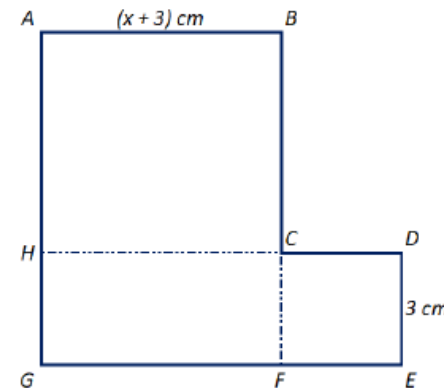
1. $x^2 + 6x + 8$
2. $x^2 + 10x + 16$
3. $x^2 + 6x + 9$
4. $x^2 + 16x + 28$
5. $x^2 - 3x + 2$
6. $x^2 - 8x + 7$
7. $x^2 + 2x - 8$
8. $x^2 - 3x - 28$

Exam questions

Expand and simplify $3(2a + 5) + 5(a - 2)$

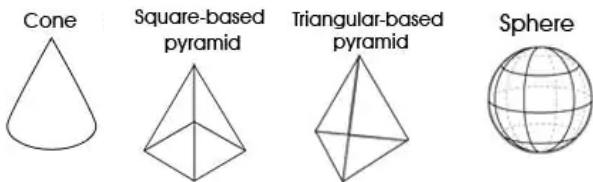
Expand and simplify
 $(x - 3)(x + 5)$

ABCH is a square
HCFG is a rectangle
CDEF is a square
They are joined to make an L-shape.



Show that the total area of the L-shape is $x^2 + 9x + 27$ cm²

Key words and definitions



Conversion

A change in the units something is given in, e.g. cm to m.

Metric units

Length: mm, cm, m, km
Mass: mg, g, kg, tonnes
Capacity: ml, cl, l

Prior Knowledge

Name all the basic parts of a circle.

Understand how to use formula.

Calculate the area of basic 2D shapes.

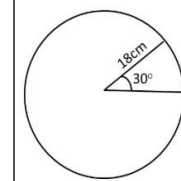
Calculate the circumference and area of a circle.

Use and apply Pythagoras' Theorem.

Perimeter and area of a sector

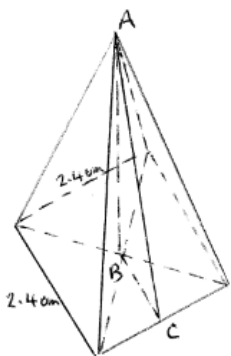
perimeter of a sector = $\frac{\theta}{360^\circ} \times 2\pi r + 2r$
 = $\frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7 + (2 \times 7)$
 = 25cm

Area of sector



Area of sector = $\frac{\theta}{360} \times \pi r^2$
 Area of sector = $\frac{30}{360} \times \pi \times 18^2$
 Area of sector = $\frac{1}{12} \times 324 \times \pi$
 Area of sector = 27π
 Area of sector = 84.82cm^2 (2dp)

Volume and surface area of a pyramid



Volume = $\frac{1}{3} \times \text{Area of the base} \times \text{height}$

$$S = B + \frac{1}{2}Pl$$

B is the area of base
P is the perimeter of base
l is the slant height

Volume of pyramid = $\frac{1}{3}(2.4 \times 2.4)(4) = 7.68 \text{ cm}^3$

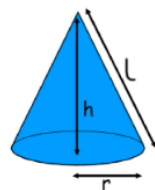
In the sketch, $AB = 4 \text{ cm}$ and $BC = 2.4/2 = 1.2 \text{ cm}$

Length $AC = \sqrt{(4^2 + 1.2^2)} = 4.176 \text{ cm}$

Hence, area of a side = $\frac{1}{2}(2.4)(4.176) = 5.01 \text{ cm}^2$

Total surface area of pyramid = $4[5.01] + (2.4)^2 = 25.81 \text{ cm}^2$

Volume and surface area of a cone



R = 5
H = 10
L = 12

Curved surface area of cone = $\pi r l$

Volume of cone = $\frac{1}{3} \pi r^2 h$

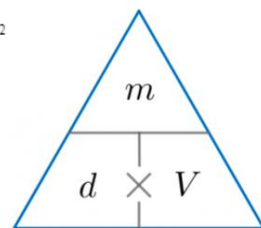
Curved surface area = $\pi r \times l = \pi \times 5 \times 12$
 = 188.4955
 = 188.50

Base area = πr^2
 = $\pi \times 5^2$
 = 78.54

Total surface area = $188.50 + 78.54 = 267.01 \text{ cm}^2$

Volume = $\frac{\pi \times 5^2 \times 10}{3} = \frac{785.3981634}{3} = 261.80 \text{ cm}^3$

Problems involving density



$m = 36 \text{ g}$ $v = 3 \text{ mL}$

density of the object = $\frac{m}{v} = \frac{36 \text{ g}}{3 \text{ mL}} = 12 \text{ g/mL}$

Volume and surface area of a sphere

Surface Area (SA) = $4 \pi r^2$

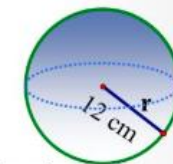
Volume (V) = $\frac{4}{3} \pi r^3$

Leave answers in terms of π

Example: Find the surface area and volume of the sphere.

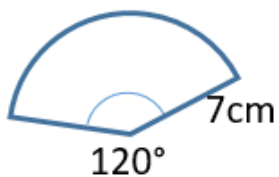
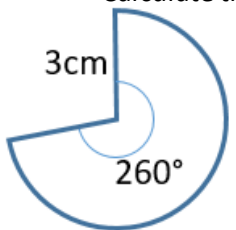
$S.A = 4 \pi (12)^2 = 576 \pi \text{ cm}^2$

$V = \frac{4}{3} \pi (12)^3 = 2304 \pi \text{ cm}^3$

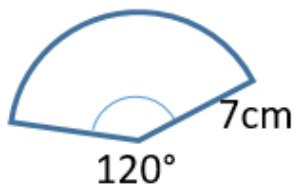
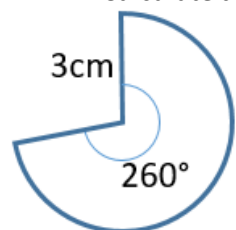


Perimeter and area of a sector

Calculate the perimeter of the sectors.

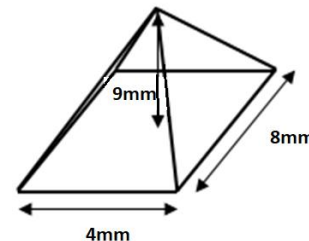
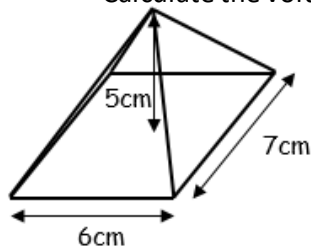


Calculate the area of the sectors.

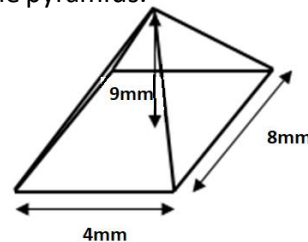
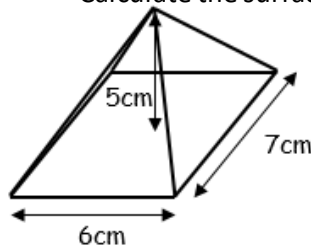


Volume and surface area of a pyramid

Calculate the volume of the pyramids.

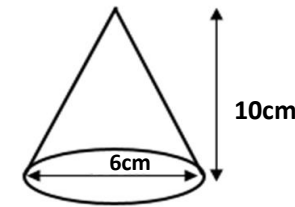
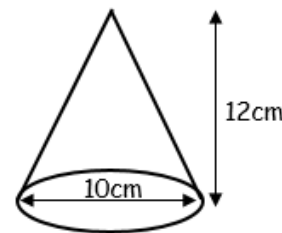


Calculate the surface area of the pyramids.

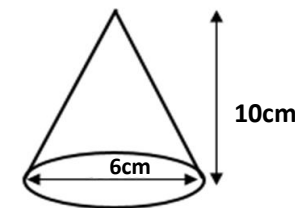
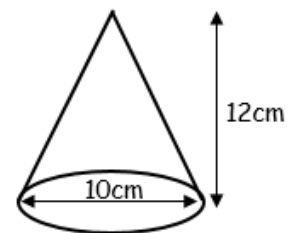


Volume and surface area of a cone

Calculate the volume of the cones.

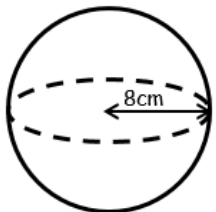


Calculate the surface area of the cones.

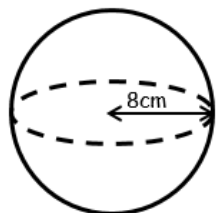


Volume and surface area of a sphere

Calculate the volume of the sphere.



Calculate the surface area of the sphere.



Problems involving density

The mass of 5 m^3 of copper is 44 800 kg. Work out the density of copper.

The density of zinc is 7130 kg/m^3 . Work out the mass of 5 m^3 of zinc.

Applying knowledge

A square based pyramid has a base area of 64 cm^2 . The distance from each of the base corners to the top of the pyramid is 10cm. Calculate the volume of the pyramid.

GCSE Foundation Topic 16 Graphs Student Knowledge Organiser

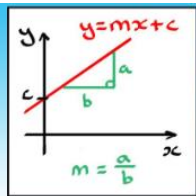
Key words and definitions

- Graph** – A diagram showing the relationship between two variables (letters), each measured along one of a pair of axes.
- Equation** – A statement that the values of two mathematical expressions are equal (indicated by the sign '=')
- Plot** – Mark out (points on) a graph.
- Parallel** – Describing two lines that never meet.
- Perpendicular** – Describing two lines that meet at right angles.
- Gradient** – The steepness of a graph
- Intercept** – The point at which a given line cuts an axis; the value of the coordinate at that point.

The Equation of a Straight Line

The Equation of a Line

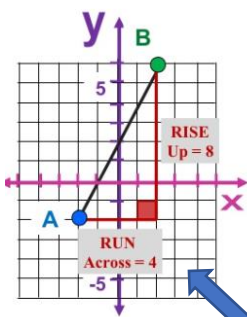
$$y = mx + c$$



m is the gradient
 c is the y-intercept

To find the equation of a line:

- 1) Find the gradient
- 2) Find the y-intercept
- 3) Write the equation of the line



The "Gradient" or "Slope" between two points is how far UP we have gone, DIVIDED BY how far we have gone ACROSS.

$$m = \frac{\text{RISE}}{\text{RUN}}$$

$$m = \frac{8}{4}$$

$$m = 2 \checkmark$$

In this example:

$$m = \frac{8}{4} = 2$$

$$c = 2$$

So the equation is $y = 2x + 2$

Midpoint and Length of a Line Segment

The Midpoint Formula is used to find the exact center point between two defined points in a line segment.

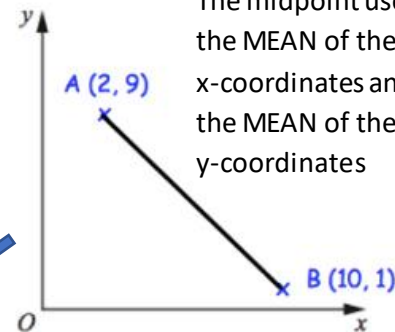
$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

x : (mean of 2 and 10)

$$\frac{2 + 10}{2} = 6$$

y : (mean of 1 and 9)

$$\frac{1 + 9}{2} = 5$$



The midpoint uses the MEAN of the x-coordinates and the MEAN of the y-coordinates

A is the point with coordinates (2, 9).
B is the point with coordinates (10, 1).

Work out the coordinates of the midpoint of the line AB.

Midpoint = (6, 5)

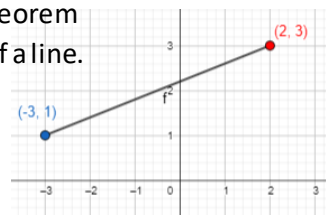
THE DISTANCE FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Use Pythagoras' Theorem to find the length of a line.

$$\text{Length} = \sqrt{(2 - (-3))^2 + (3 - 1)^2}$$

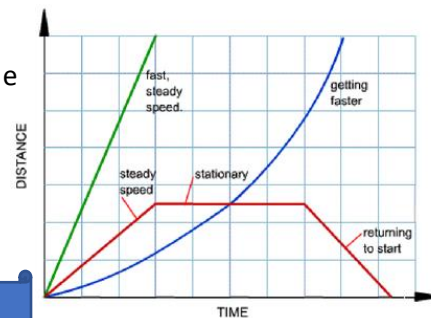
$$= \sqrt{5^2 + 2^2} = \sqrt{29} = 5.39$$



Real Life Graphs

Graphs can be used to model real life situations, for example a car journey.

When using a distance-time graph, the following parts of a graph have to be considered.



The gradient of the graph shows the speed of the car.

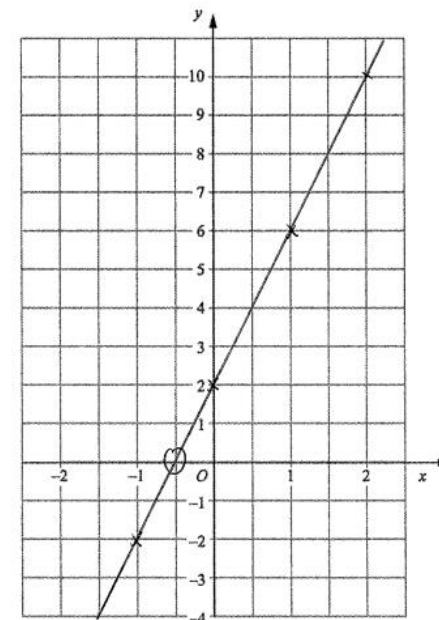
Drawing Straight Line Graphs

- (a) Complete the table of values for $y = 4x + 2$. To find y , multiply x by 4 and add 2.

x	-1	0	1	2
y	-2	2	6	10

$$2 \times 4 + 2 = 10$$

- (b) On the grid, draw the graph of $y = 4x + 2$.



Plot the points:
(-1, -2)
(0, 2)
(1, 6)
(2, 10)
And join with a straight line.

Drawing Graphs

(a) $y = 3x + 3$

x	-2	-1	0	1	2
y					

(b) $y = x + 9$

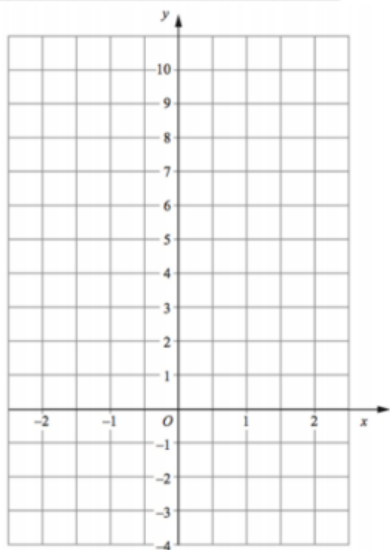
x	-2	-1	0	1	2
y					

(c) $y = \frac{1}{2}x + 1$

x	-2	-1	0	1	2	3	4
y							

(d) $y = -2x + 5$

x	-1	0	1	2	3
y					



Complete the table of values for each equation, and plot the line on the axes below.

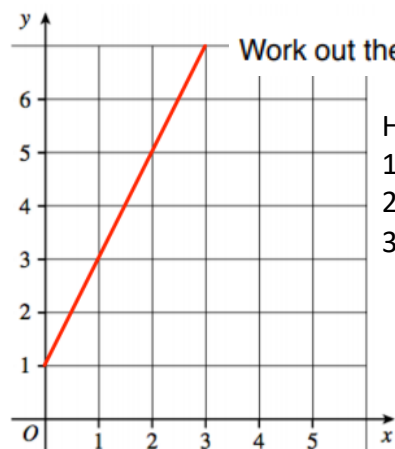
The Equation of a Straight Line

1. A line has equation $y = 3x + 4$

(a) Write down the gradient of the line

(b) Write down the y-intercept of the line

2. A straight line L is shown on the grid.

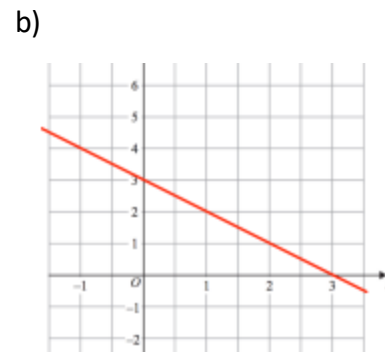
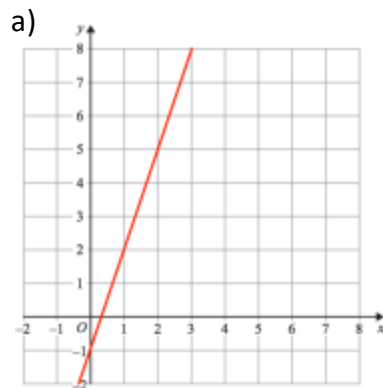


Work out the equation of line L

Hint:

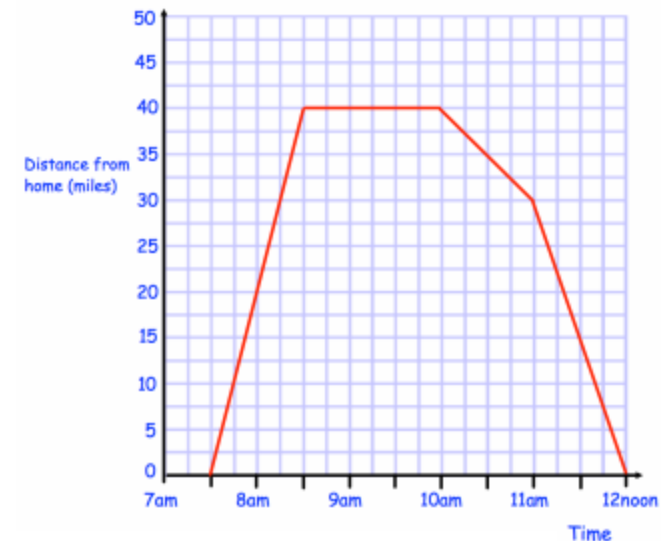
- 1) Find the gradient
- 2) Find the y-intercept
- 3) Write out the equation of the line

3. Work out the equation of the following lines:



Distance-Time Graphs

Emma travelled to her Grandmother's house and back. The distance-time graph shows information about her journey



- (a) What time did Emma begin her journey?
- (b) How far was Emma from home at 8am?
- (c) How long did Emma stay at her Grandmother's house?
- (d) What time did Emma leave her Grandmother's house?
- (e) How far was Emma from home at 11:45?
- (f) How far did Emma travel in total?
- (g) What was Emma's speed from her house to her Grandmother's house?

GCSE Foundation Topic 17 Constructions Student Knowledge Organiser

Key words and definitions

Constructions- Mathematical drawings that use only a pencil, ruler, compass and protractor.

Perpendicular- At a 90° angle from a line, surface or plane.

Bisector- Cuts somethings in half.

Congruent- Two shapes are congruent if they are mathematically identical

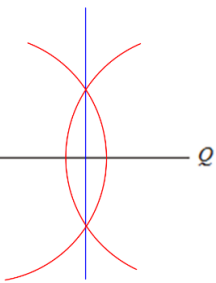
Parallel- Two lines that remain the same distance apart at all times.

Vertex- Point/Corner

Perpendicular bisectors

Start with a line segment

- 1) Place you compass on one end of the line and stretch it wider than half way across.
- 2) Without changing the compasses width make an arc across the line (red arcs on diagram).
- 3) Place your compass on the other end of the line segment and without changing the width of the compass make another arc across the line (red arcs on diagram).
- 4) Draw a line that crosses though where both arcs intersect. (Blue line of diagram)



Constructing Angles

Constructions can be used to draw 30° , 45° , 60° and 90° . To remind yourself how to do these refer to the Hegarty Maths videos:

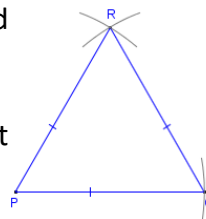
664- Construct a 90° or 45° angle.

665- Construct a 60° , 30° or 120° angle.

Constructing an equilateral triangle.

Start with a line segment that is the desired side length for your triangle.

- 1) Place you compass on one end of the line and stretch it to be the length of the line.
- 2) Make an arc above the line.
- 3) Keeping the compass the same length place it on the opposite side of the line and make another arc above.
- 4) Where the two arcs cross is the third corner of your triangle. Join it up with the other two.



Constructing other triangles

There are many other ways to construct triangles. Including:

SSS- Using three side lengths.

SAS- Using an angle between two sides.

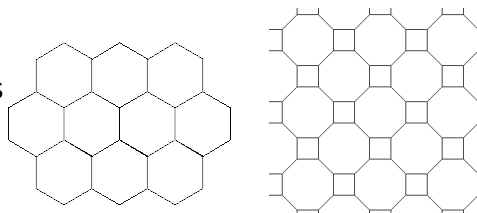
ASA- Using a side between two angles.

Please refer to the Hegarty video for instruction on drawing these:

683- Constructing triangles.

Tessellations

Shapes tessellate when they fit together to make a pattern without leaving any gaps. This is possible whenever the shape's angles add up to 360° .

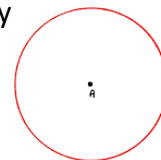


Loci

A locus (plural loci) is a set of points that satisfy a certain condition.

Example 1: Draw the set of point that are 5cm away from a point.

To do this you would just draw a circle of radius 5cm around that point.



Example 2: Draw the set of points that are 2cm away from a line.

To do this you would draw two parallel lines above and below your that are 2cm away. At the end of those lines you would join them by a semi-circle that has its centre at the end of your original line



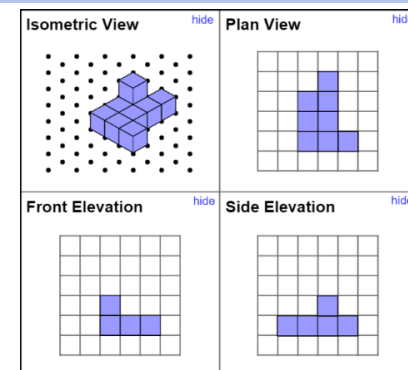
Plans and Elevations

Plans and elevations are 2D drawings of 2D shapes from different angles.

The **Plan** of a shape shows the shape from above.

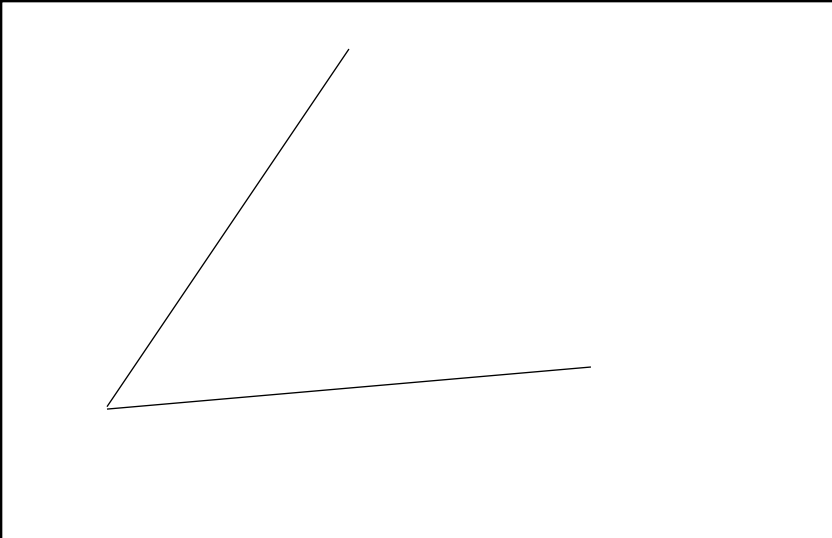
The **Front Elevation** shows the shape from the front.

The **Side Elevation** shows the image from the side.

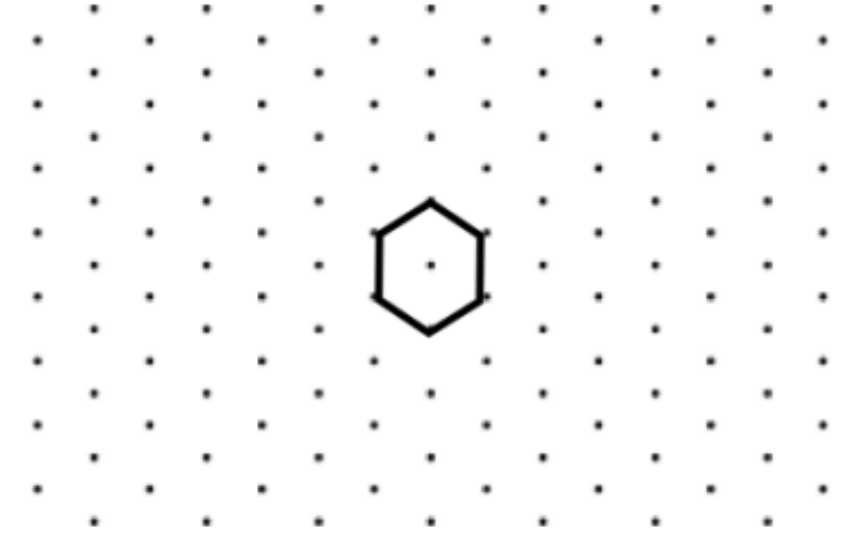


GCSE Foundation Topic 17 Constructions Student Knowledge Organiser

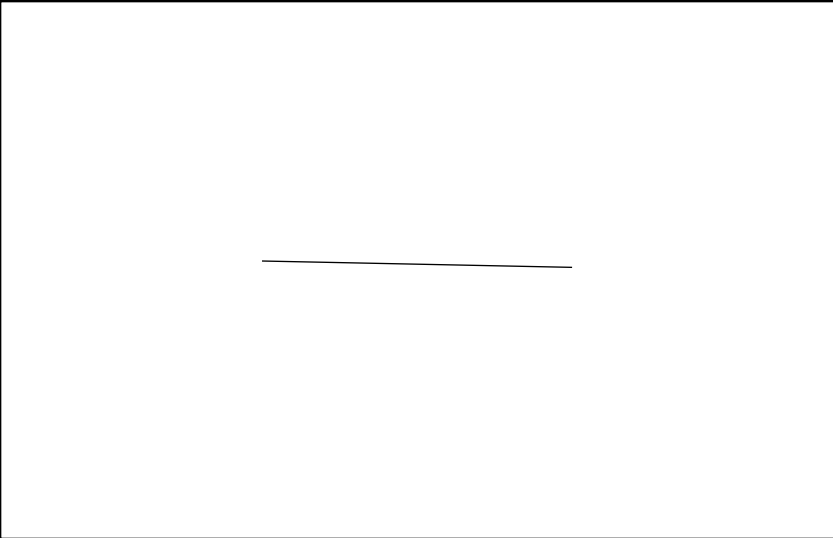
Using a compass and ruler bisect the angle below:



Tessellate the shape below, draw at least five shapes:



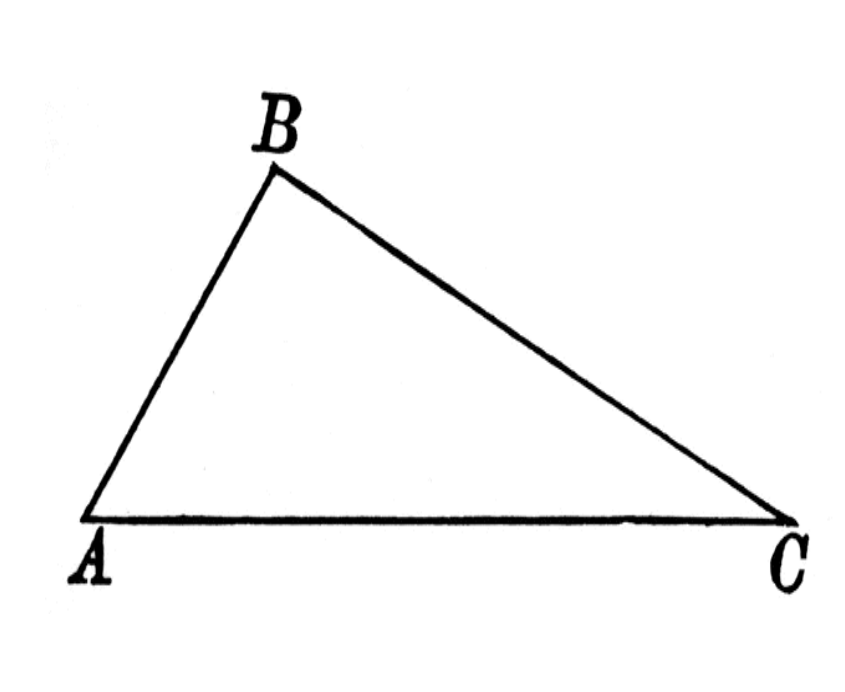
Draw all the points that are 2cm from the line below



Construct an equilateral triangle with side length 5cm:



Shade the area that's less than 2cm from A



Draw the plan and elevations for the shape below.

<p>Isometric View hide</p> <p>A 3D shape composed of five unit cubes is shown on a dot grid. It consists of a base of three cubes in a row, with one cube stacked on top of the middle cube, and another cube attached to the side of the middle cube.</p>	<p>Plan View show</p> <p>A 5x5 grid for drawing the plan view of the shape.</p>
<p>Front Elevation show</p> <p>A 5x5 grid for drawing the front elevation of the shape.</p>	<p>Side Elevation show</p> <p>A 5x5 grid for drawing the side elevation of the shape.</p>

Key words and definitions

Denominator - The number below the line in a fraction.

Denotes how many parts the whole is divided into.

Least Common Denominator - The least common multiple of the denominators of two or more fractions. That is, the lowest number which is a multiple of both denominators.

Lowest Terms - A fraction whose numerator and denominator have no common factors is in *lowest terms*.

Mixed Number - A number composed of a whole number and a fraction.

Fractions

Dividing

$$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{2 \times 3}{5 \times 2} = \frac{6}{10} = \frac{3}{5}$$

take the reciprocal of the divisor

Adding

$$\frac{1}{5} + \frac{3}{7} = \frac{1 \times 7}{5 \times 7} + \frac{3 \times 5}{7 \times 5} = \frac{7}{35} + \frac{15}{35}$$

Add the Numerators

$$\frac{7}{35} + \frac{15}{35} = \frac{22}{35}$$

Multiplying

$$\frac{2}{5} \times \frac{6}{7} = \frac{2 \times 6}{5 \times 7} = \frac{12}{35}$$

$$1 \frac{3}{4} \times 2 \frac{1}{2} = ?$$

$1 \times 4 + 3 = 7$
 $2 \times 2 + 1 = 5$

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4 \frac{3}{8}$$

Reverse Percentages

Example

A shop offers 30% off everything in a sale. The sale price of a pair of designer shoes is £84. Calculate the cost of the shoes before the sale.

$$\text{Sale of } 30\% \rightarrow (100\% - 30\% = 70\%)$$

$$70\% \text{ of the cost of the shoes} = £84$$

$$1\% \rightarrow 84 \div 70 = £1.20$$

$$100\% \rightarrow £1.20 \times 100 = £120$$

The cost of the shoes before the sale was £120.

Increase/Decrease by a percentage using a multiplier

Example

What is the **multiplier** for a 15% increase?

A 15% increase would mean that the overall percentage

would be: $100\% + 15\% = 115\%$

115% as a decimal = $115 \div 100 = 1.15$

Increase £48 by 15%

$$£48 \times 1.15 = £55.20$$

Powers

$$\left(\frac{3}{4}\right)^2 = \frac{3}{4} \cdot \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

Standard Form

To convert a number into **standard form**, split the number into two parts - a number between 1 and 10 multiplied by a **power** of 10.

Example

Write 81 900 000 in standard form:

$$81\,900\,000 = 8.19 \times 10\,000\,000 = 8.19 \times 10^7$$

Check: It's 10^7 because the decimal point has been moved 7 places to the left to get the number to be 8.19

Example

Write 0.000 001 2 in standard form:

$$0.000\,001\,2 = 1.2 \times 0.000\,001 = 1.2 \times 10^{-6}$$

It's 10^{-6} because the decimal point has been moved 6 places to the right to get the number to be 1.2

Fractions

1) $3\frac{2}{5} + \frac{14}{15}$	2) $2\frac{1}{2} + 4\frac{4}{5}$
3) $5\frac{6}{7} - 2\frac{1}{5}$	4) $5 - 2\frac{3}{7}$
5) $2\frac{7}{9} - \frac{2}{5}$	6) $\frac{3}{10} \times \frac{14}{15}$
7) $1\frac{1}{4} \times 1\frac{2}{5}$	8) $1\frac{1}{4} \times 2\frac{2}{7} \times 1\frac{2}{5}$
9) $2\frac{3}{4} \times \frac{2}{5} \times 1\frac{4}{11}$	10) $18 \div \frac{1}{4}$
11) $\frac{9}{10} \div \frac{3}{5}$	12) $1\frac{2}{5} \div \frac{14}{15}$

Percentages

- If Bobby went to the shop and there was a 20% sale. He was going to buy a top for £20.
How much does Bobby save?
What is the new price of the top?
- Apple is having a 20% off sale. I bought my new Ipod for £40, how much was it originally?
- A special bottle of coke contains 10% more than a normal bottle. The special bottle contains 660 ml. How much does the normal bottle contain?
- Katie gets a 20% pay rise. Her new wage is £264 per week. What was her wage before the pay rise?

Standard Form

1. Write the following numbers in standard form:

- 3560000
- 258300
- 3100000
- 45000000
- 7990000
- 101000
- 23450
- 465600
- 24500000

2. Write the following numbers in standard form

- 0.000432
- 0.245
- 0.00753
- 0.0234
- 0.00451
- 0.00405
- 0.005714
- 0.0004013
- 0.004487
- 0.0000012034

- Work out $(8 \times 10^4) \times (8 \times 10^4)$
- Work out $(2.7 \times 10^6) \div (3 \times 10^2)$
- Work out $(3.2 \times 10^5) \div (4 \times 10)$
- Work out $(1.8 \times 10^7) \div (3 \times 10^3)$
- Work out $(4.8 \times 10^6) \div (8 \times 10^3)$
- Work out $(1.8 \times 10^{10}) \div (9 \times 10^4)$
- Work out $(9 \times 10^5) \times (6 \times 10^5)$
- Work out $(9 \times 10^2) \times (8 \times 10^5)$

Key words and definitions

Quadratic graph - The graph of a quadratic function is a parabola whose line of symmetry is parallel to the y-axis.

Parabola - a symmetrical curve.

Gradient - Another word for "slope". The higher the gradient of a graph at a point, the steeper the line is at that point. A negative gradient means that the line slopes downwards.

Y intercept- The point where a line crosses the y axis.

Roots of a quadratic graph- These are the x-intercepts. It is where $y = 0$ so, $ax^2 + bx + c = 0$.

Quadratic Expressions

$$ax^2 + bx + c$$

$c = y$ intercept (where the parabola CUTS the y axis)

Parabolas are symmetrical

When a is positive



When a is negative



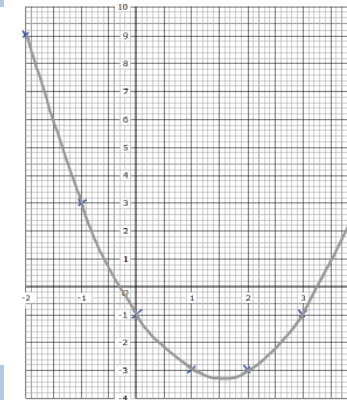
Plotting a quadratic graph

Complete the table of values for $y = x^2 - 3x - 1$

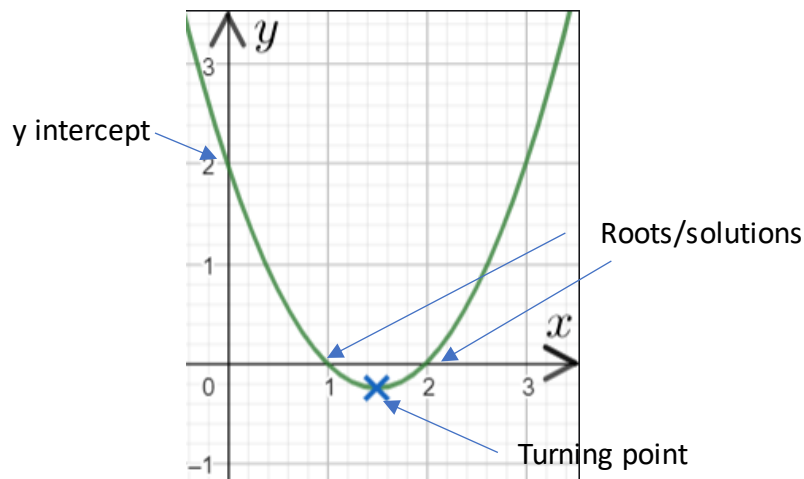
x	-2	-1	0	1	2	3	4
y	9	3	-1	-3	-3	-1	3

Substitute the x values into the equation to find the y coordinates. Be careful with negative numbers! Remember -3^2 is 9.

On the grid, draw the graph of $y = x^2 - 3x - 1$ for values of x from -2 to 4.



Key points on a quadratic graph



Equations of a straight line

The equation of a straight line graph is in the form:

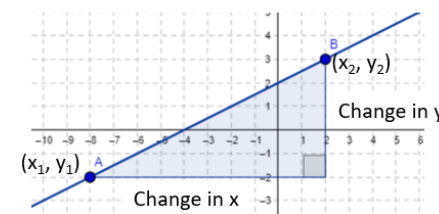
$$y = mx + c$$

where $m = \text{gradient}$ $c = y$ intercept

Example. For $y = 2x + 3$ the line has a **gradient of 2** and a **y intercept of +3**

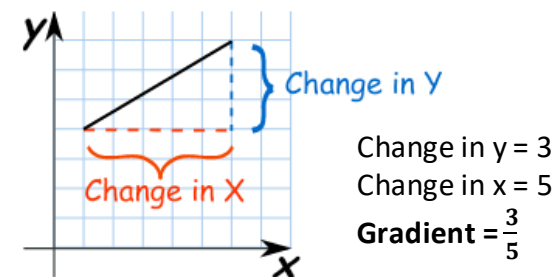
Finding the gradient

Gradient of a Straight Line



$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in y}}{\text{Change in x}}$$

Example



Parallel and perpendicular lines

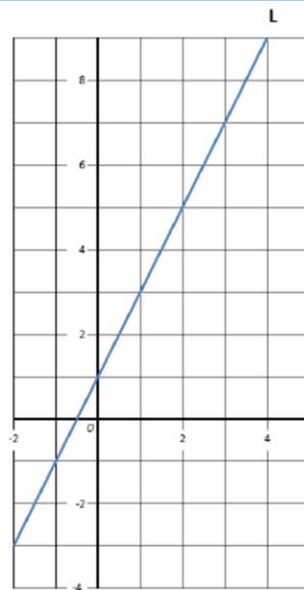
Lines that have **the same gradient are parallel**
Eg. $y = 3x + 4$ is a parallel to the line $y = 3x - 4$
They both have a gradient of **3** so are **parallel**.

If two lines **are perpendicular**, then **their gradients will multiply together to give -1**. Find the equation of a line perpendicular to $y = 3 - 5x$. This line has gradient -5 . A **perpendicular line will have to have a gradient of 1/5**, because then $(-5) \times (1/5) = -1$.

Gradients and Parallel lines

- The equation of a straight line is $y = 4x + 5$, what is the gradient of the line and the y intercept?
- The equation of a straight line is $y = 6x + 3$, give the equation of 2 lines that will be parallel with this line.
- Which line would be steeper;
 $y = 0.5x + 2$ or $y = 2x + 2$?
- The equation of a line is $y = 5x - 3$ What is the equation of the line perpendicular to this line?

$Y = mx + c$

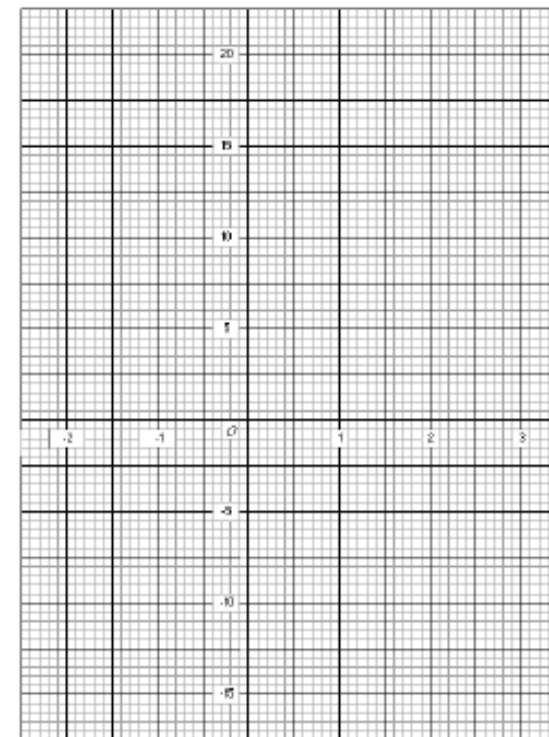


Drawing quadratic and cubic graphs

Complete the table of values for $y = x^3 - 7$

x	-2	-1	0	1	2	3
Y		-8				20

On the grid, draw the graph of $y = x^3 - 7$ for values of x from -2 to 3.



Exam questions

L is a straight line.

The gradient of L is 4

L passes through the point (0, 2)

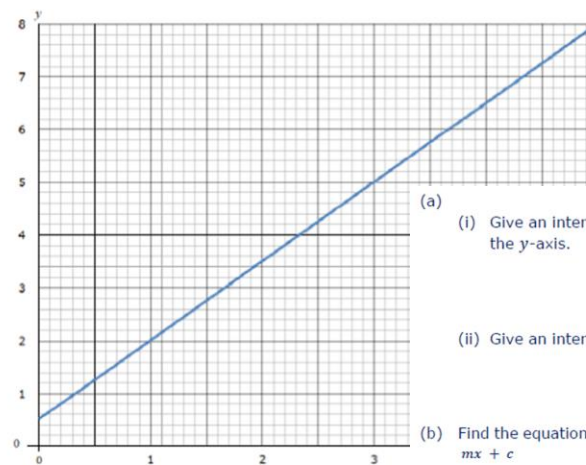
Write down the equation of the straight line L.

The equation of the line L_1 is $y = 3x - 2$

The equation of the line L_2 is $3y - 9x + 5 = 0$

Show that these two lines are parallel.

Find the equation for the straight line L.



- (a)
- Give an interpretation of the intercept of the graph on the y-axis.
 - Give an interpretation of the gradient of the graph.
- (b) Find the equation of the straight line in the form $y = mx + c$

Phone calls cost £y for x minutes.

The graph gives the values of y for values of x from 0 to 5

Draw the graph $y = x^2 + 2x - 3$

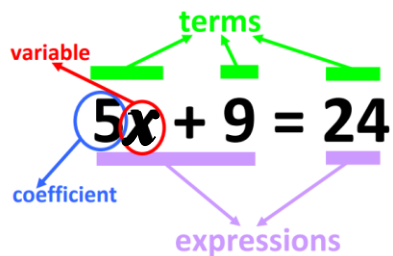
For y values between -3 and 3.

Mark on your graph the turning point, the line of symmetry and the roots of the equation when $x^2 + 2x - 3 = 0$

GCSE Foundation Topic 20 Equations 2 Student Knowledge Organiser

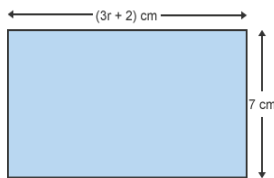
Key words and definitions

Word	Definition
Variable	A symbol for an unknown value. Usually a letter, such as a , x or y , is the symbol used for a variable.
Constant	A number on its own
Coefficient	A number that is multiplied by a variable. Example: $8y$ means 8 times y ; 8 is the coefficient, and y is the variable.
Operator	A symbol (+, \times , $-$, or \div) representing a mathematical operation
Term	Either a single number, a variable, or numbers and/or variables multiplied together Examples: 4 45 x abc $5w$ $20mn$
Expression	A term or a combination of terms and operators Examples: 2 $2x$ $2x+7$ y $y-3$ $7w+3$ $8ab+9$ $5xyz$
Equation	A mathematical sentence stating that two expressions are equal



Forming and Solving Equations

The area of this rectangle is 56 cm^2 . Find the value of r .



Area of a rectangle = $\text{base} \times \text{height}$. This means $3r + 2$ will all be multiplied by 7. To show this in algebra, use a bracket for $3r + 2$ to show that both terms are being multiplied by 7.

7 multiplied by $(3r + 2)$ can be written as $7(3r + 2)$ as multiplication signs are not used in algebra.

Area = $\text{base} \times \text{height}$

$$\text{Area} = 7(3r + 2)$$

The area of the rectangle has been given in the question as 56 cm^2 :

$$56 = 7(3r + 2)$$

Expand the bracket:

$$56 = 7 \times 3r + 7 \times 2$$

$$56 = 21r + 14$$

Isolate $21r$ by subtracting 14 from both sides:

$$56 - 14 = 21r + 14 - 14$$

$$42 = 21r$$

Isolate r by dividing both sides by 21:

$$42 \div 21 = 21r \div 21$$

$$2 = r$$

Rearranging Formulae

The **subject** of a formula is the variable that is being worked out. It can be recognised as the letter on its own on one side of the equals sign.

For example, in the formula for the area of a rectangle $A = bh$ ($\text{area} = \text{base} \times \text{height}$), the subject of the formula is A .

Rearrange the formula $v = u + at$ to make t the subject of the formula.

$$v = u + at$$

$$-u \quad -u$$

$$v - u = at$$

$$\div a \quad \div a$$

$$\frac{v - u}{a} = t$$

The letter t is now isolated, so t is now the subject of the formula.

Rearrange the formula $T = 2\pi\sqrt{\frac{L}{G}}$ to make L the subject.

Firstly, isolate the root: Now 'square' both sides: Lastly, multiply by G :

$$\frac{T}{2\pi} = \frac{2\pi\sqrt{\frac{L}{G}}}{2\pi}$$

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{G}}\right)^2$$

$$\frac{\left(\frac{T}{2\pi}\right)^2}{\times G} = \frac{L}{\times G}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{G}}$$

$$G\left(\frac{T}{2\pi}\right)^2 = L$$

Solving Simultaneous Equations

Solve the following simultaneous equations:

$$3x + y = 11$$

$$2x + y = 8$$

First, identify which unknown has the same coefficient. In this example this is the letter y , which has a coefficient of 1 in each equation.

Either add or subtract the two equations from each other to eliminate the letter y . In this example the equations will need to be subtracted from each other as $y - y = 0$.

$$3x + y = 11$$

$$- \quad - \quad -$$

$$2x + y = 8$$

$$= \quad = \quad =$$

$$x = 3$$

The value of x can now be **substituted** into either equation to find the value of y .

Substitute $x = 3$ into either $3x + y = 11$ or $2x + y = 8$.

$$3x + y = 11 \text{ when } x = 3$$

Substitute $x = 3$:

$$3 \times 3 + y = 11$$

$$9 + y = 11$$

Find the value of y using **inverse operations** to **solve equations**.

The inverse of adding 9 is subtracting 9, so subtract 9 from each side:

$$9 + y - 9 = 11 - 9$$

$$y = 2$$

Check the answers by substituting both values into the other original equation. If the equation balances, then the answers are correct:

$$2x + y = 8 \text{ when } x = 3 \text{ and } y = 2.$$

$$2x + y = 2 \times 3 + 2 = 6 + 2 = 8.$$

In examples like this, one or both equations must be multiplied to create a common coefficient.

$$3a + 2b = 17$$

$$4a - b = 30$$

Multiply the bottom equation to create a common coefficient of $2b$.

$$3a + 2b = 17$$

$$8a - 2b = 60$$

These equations can now be used to find the values of a and b .

The signs in front of the common coefficients are different, so the equations should be added together:

$$3a + 2b = 17$$

$$+ \quad + \quad +$$

$$8a - 2b = 60$$

$$= \quad = \quad =$$

$$11a = 77$$

$$\div 11 \quad \div 11$$

$$a = 7$$

Substitute the value of a into one of the original equations to find the value of b .

$$3a + 2b = 17 \text{ (when } a = 7)$$

Substitute $a = 7$:

$$3 \times 7 + 2b = 17$$

$$21 + 2b = 17$$

Solve the equation by using **inverse operations**. The opposite of $+21$ is -21 . Subtract 21 from both sides of the equation:

$$2b = -4$$

$$b = -2$$

Check the answers:

$$4a - b = 30 \text{ when } a = 7 \text{ and } b = -2.$$

$$4 \times 7 - -2 = 30$$

Solving Linear Equations

Solve the equation $4y + 5 = -3$.

$$4y + 5 = -3$$

Subtract 5 from each side:

$$4y + 5 - 5 = -3 - 5$$

Simplify:

$$4y = -8$$

Get y by itself by dividing both sides by 4:

$$4y \div 4 = -8 \div 4$$

$$y = -2$$

Solve the equation $5(2c - 3) = 19$.

Expand the bracket:

$$5 \times 2c - 5 \times 3 = 19$$

$$10c - 15 = 19$$

Isolate $10c$ by adding 15 to each side:

$$10c - 15 + 15 = 19 + 15$$

$$10c = 34$$

Isolate c by dividing by 10:

$$10c \div 10 = 34 \div 10$$

$$c = \frac{34}{10} = \frac{17}{5} \text{ or } 3.4$$

Solving linear equations

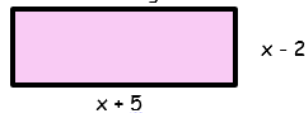
- | | |
|---------------------------|----------------------------|
| (a) $2x + 3 = 9$ | (b) $3w - 1 = 14$ |
| (d) $5x + 20 = 35$ | (e) $6c - 12 = 48$ |
| (g) $7w + 13 = 90$ | (h) $12p - 18 = 30$ |
| (i) $10a + 40 = 100$ | (k) $9x - 24 = 84$ |
| (m) $6x - 19 = 5$ | (n) $3w + 4 = 43$ |
| (p) $\frac{c}{2} - 4 = 6$ | (q) $\frac{x}{10} + 3 = 9$ |

- | | |
|------------------------|------------------------|
| (a) $4x + 1 = 2x + 7$ | (b) $5x + 4 = 3x + 16$ |
| (d) $7x + 1 = 2x + 46$ | (e) $6x - 3 = 2x + 13$ |
| (g) $2x + 21 = 4x + 5$ | (h) $x + 2 = 5x - 2$ |
| (i) $5x + 2 = 16 - 2x$ | (k) $3x - 1 = 23 - x$ |

Forming and solving equations

1) Ahmad is twice as old as Bobby. John is 7 years younger than Ahmad. If the sum of their age is 38, how old are the three boys?

2) The perimeter of the rectangle below is 42cm. Calculate the lengths of the sides by forming an equation and solving it.



- 3) A garden measures p metres by $3p + 2$ metres.
- Write an expression that describes the perimeter of the garden.
 - The garden has a perimeter of 76 metres. Write an equation to show this.
 - Solve your equation to find the value of p .

Simultaneous equations

Solve the following simultaneous equations by using elimination.

- | | | |
|--------------------------------------|---------------------------------------|--|
| (j) $2x - 4y = 10$
$2x + 3y = 24$ | (k) $5x - 2y = 120$
$5x + y = 165$ | (l) $x - 2y = 8$
$x - 3y = 3$ |
| (m) $3x + 2y = 54$
$2x - 2y = 16$ | (n) $7x - 4y = 80$
$3x - 4y = -80$ | (o) $5x - 2y = -23$
$5x - 6y = -39$ |
| (a) $3x + 2y = 23$
$2x - y = 6$ | (b) $3x - 3y = 9$
$2x + y = 12$ | (c) $4x + 2y = 34$
$3x + y = 21$ |
| (d) $9x - 4y = 59$
$2x - y = 12$ | (e) $2x + 8y = 43$
$x + 3y = 18$ | (f) $6x + 3y = 45$
$2x - 2y = 12$ |

Applying Knowledge

1. Solve $4(x - 3) = 7x - 10$
Show clear algebraic working.

2.

Here is a rectangle.

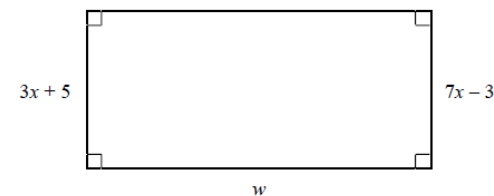


Diagram NOT accurately drawn

All measurements are in centimetres.
The area of the rectangle is 242 cm^2 .
Find the value of w .

3. HINT... Think simultaneous equations!!

Five adult tickets and three child tickets for a movie cost £58.
Two adult tickets and eight child tickets for a movie cost £47.
Find the cost of each type of ticket.

Rearranging formulae

Make x the subject of the following formulae

- | | | |
|---------------------------|---------------------------|---------------------------------|
| (a) $4x + c = w$ | (b) $dx - t = 8$ | (c) $x^2 + 3 = h$ |
| (d) $2x + 2y = P$ | (e) $s = x^2 - 3$ | (f) $y = xz + s$ |
| (g) $\frac{x}{n} + 2 = w$ | (h) $\frac{x}{6} - 5 = w$ | (i) $\frac{x+3}{c} = h$ |
| (j) $3y = 4x + 1$ | (k) $x^2 + a = v$ | (l) $x^3 - 4 = 5y$ |
| (m) $\frac{x+t}{m} = 2c$ | (n) $\frac{w+x}{u} = 3z$ | (o) $A = \pi x^2$ |
| (p) $A = \frac{1}{2}bx$ | (q) $V = abx$ | (r) $v^2 = u^2 + 2ax$ |
| (s) $\frac{a+b}{x} = r$ | (t) $\frac{5cx}{b} = a$ | (u) $\sqrt[3]{\frac{x}{k}} = w$ |

GCSE Foundation Topic 22 Vectors Student Knowledge Organiser

Key words and definitions

Magnitude – the length of a vector

Vector – a quantity that is described by a magnitude and a direction.

Scalar – a quantity that is described by a magnitude (or numerical value) alone.

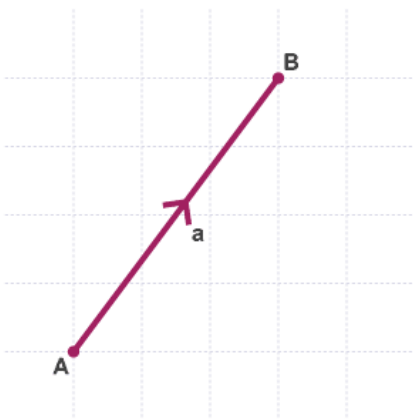
Direction – the direction along which it acts.

Scalar Multiple – the amount by which a vector's magnitude is changed.

Parallel – Vectors acting in the same direction will be parallel (side-by-side).

Column Vectors

A vector between two points A and B is described as: \overrightarrow{AB} , a or \underline{a} .



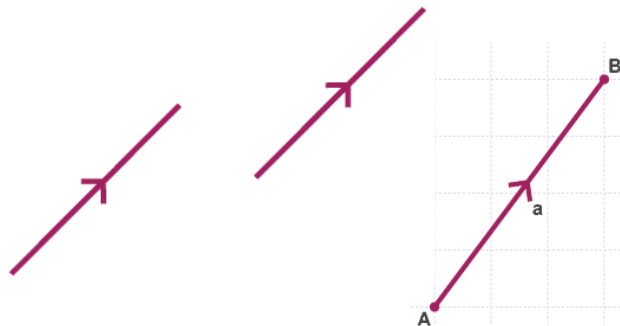
The vector can also be represented by the **column vector** $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

The top number tells you how many spaces or units to move in the positive x -direction and the bottom number is how many to move in the positive y -direction.

Vectors are equal if they have the same magnitude and direction regardless of where they are.

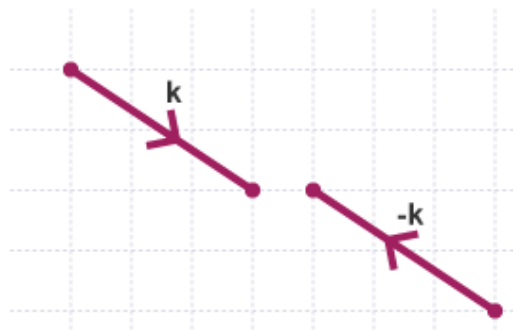
Drawing Vectors

A vector can be represented by a **line segment** labelled with an arrow.



A vector between two points A and B is described as: \overrightarrow{AB} , a or \underline{a} .

A negative vector has the same magnitude but the opposite direction.



Vectors can be multiplied by a **scalar** which changes the size of the vector but not the direction.

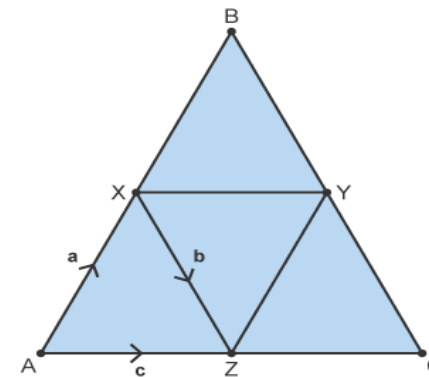
$$k = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

The vector $2k$ is twice as long as the vector k . Double each number in k to get $2k$.

Vectors around a Shape

Example

Write, in terms of a , b and c , the vectors \overrightarrow{ZY} , \overrightarrow{YC} , \overrightarrow{ZA} and \overrightarrow{BX} .



$$\overrightarrow{ZY} = a$$

\overrightarrow{ZY} and \overrightarrow{AX} are equal vectors, they have the same magnitude and direction.

$$\overrightarrow{YC} = b$$

\overrightarrow{YC} and \overrightarrow{XZ} are equal vectors, they have the same magnitude and direction.

$$\overrightarrow{ZA} = -c$$

\overrightarrow{ZA} has the same magnitude as \overrightarrow{AZ} but the opposite direction.

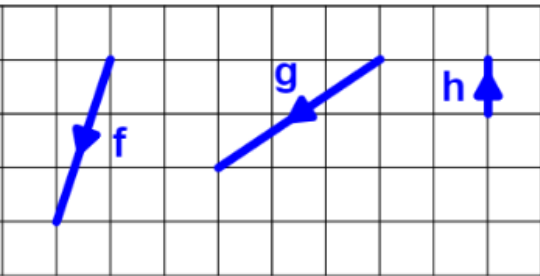
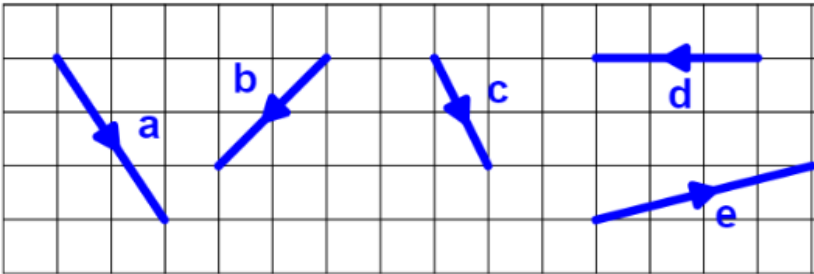
$$\overrightarrow{BX} = -a$$

\overrightarrow{BX} has the same magnitude as \overrightarrow{AX} but the opposite direction.

GCSE Foundation Topic 22 Vectors Student Knowledge Organiser

Column Vectors

Write a column vector for each vector shown in the diagram.



Given that $\mathbf{p} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, write a column vector for:

- a) $3\mathbf{p}$ b) $-\mathbf{p}$ c) $-2\mathbf{p}$ d) $\frac{1}{3}\mathbf{p}$ e) $-\frac{2}{3}\mathbf{p}$ f) $0.1\mathbf{p}$

Which of the following are parallel to the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$? Select all that apply.

- a) $\begin{pmatrix} 6 \\ 15 \end{pmatrix}$ b) $\begin{pmatrix} -6 \\ 15 \end{pmatrix}$ c) $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ d) $\begin{pmatrix} 4 \\ -10 \end{pmatrix}$ e) $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$ f) $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

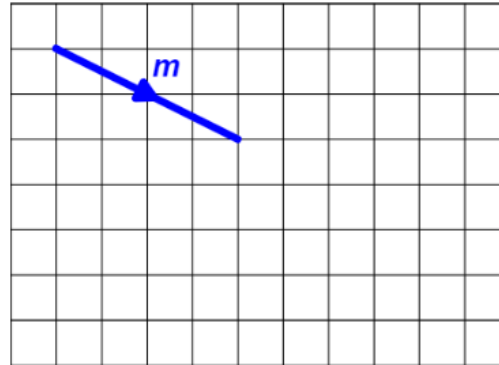
Which of the following are parallel to the vector $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$? Select all that apply.

- a) $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$ b) $\begin{pmatrix} -6 \\ 9 \end{pmatrix}$ c) $\begin{pmatrix} 9 \\ 6 \end{pmatrix}$ d) $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ e) $\begin{pmatrix} -6 \\ -9 \end{pmatrix}$ f) $\begin{pmatrix} 9 \\ 12 \end{pmatrix}$

Drawing Vectors

The vector \mathbf{m} is shown on the grid. Draw each of these vectors on the same grid:

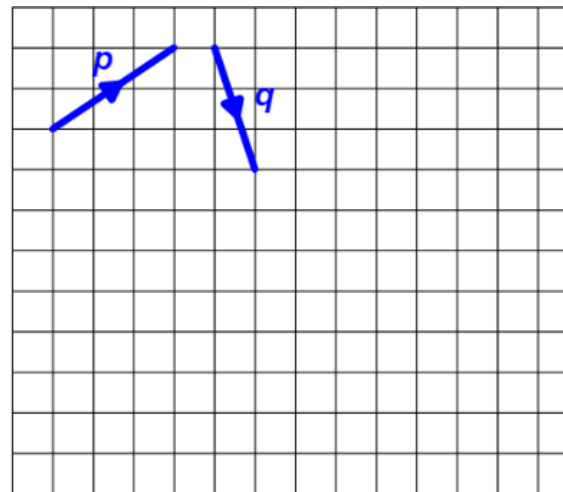
- a) $2\mathbf{m}$ b) $-\mathbf{m}$
c) $-2\mathbf{m}$ d) $\frac{1}{2}\mathbf{m}$



The vectors \mathbf{a} and \mathbf{b} are shown on the square grid.

Draw the vectors:

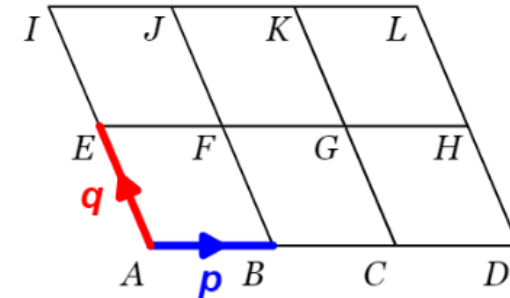
- a) $-2\mathbf{p}$
b) $\mathbf{p} + \mathbf{q}$
c) $\mathbf{p} - \mathbf{q}$



Vectors around a shape

The grid contains six congruent parallelograms. $\vec{AB} = \mathbf{p}$ and $\vec{AE} = \mathbf{q}$. Write in terms of \mathbf{p} or \mathbf{q} :

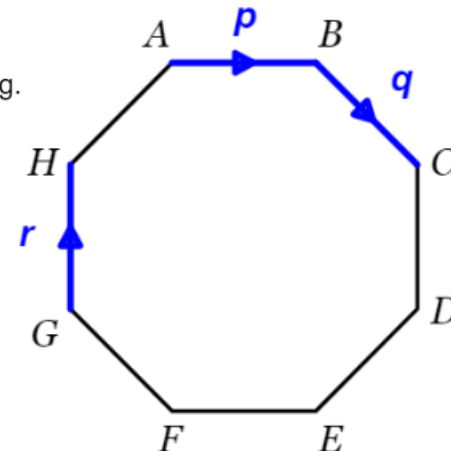
- a) \vec{FJ} b) \vec{KL} c) \vec{GC}
d) \vec{AI} e) \vec{LD} f) \vec{HE}



$ABCDEFGH$ is a regular octagon. $\vec{AB} = \mathbf{p}$, $\vec{BC} = \mathbf{q}$, and $\vec{GH} = \mathbf{r}$.

- a) Write in terms of \mathbf{p} , \mathbf{q} or \mathbf{r} :
i) \vec{GF} ii) \vec{CD}
iii) \vec{FG} iv) \vec{FE}

- b) Sam writes: $\vec{AH} = \mathbf{q}$. Explain why Sam is wrong.



How do I revise?**1. Do you know your formulae?**

Look, Cover, Write, Check...

Flash cards... test yourself every few days...

Write down some examples of how you would use each one...

2. Do you know your Key Facts?

Flash cards... test yourself every few days...

Write down some examples of how you would use each one...

Test yourself regularly...

3. Do you know each topic? Can you use to correct notation?

Read your knowledge organisers carefully

Hegarty Maths: Watch videos and complete tasks

– Make sure you are comfortable with it

Complete the practice questions in your KO

4. Can you apply your skills to test questions ?

Practice exam questions on one topic – show

your working clearly – check your answers

5. Can you complete exam papers?

Paper in class – Can you complete mixed

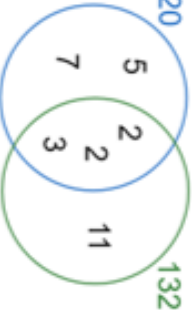


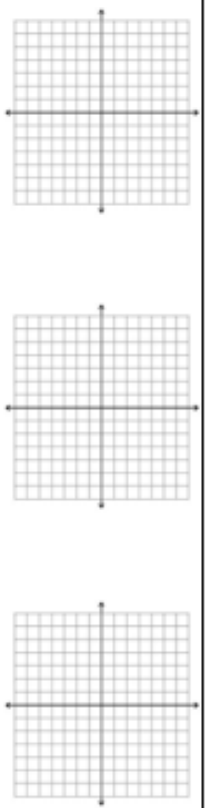
questions? Can you complete in time? What do

you know? Which topics to you need to work on?

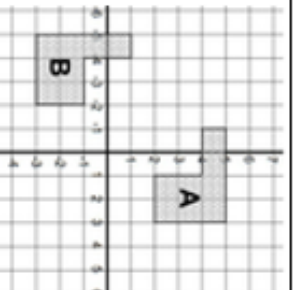
Papers at home – complete all of the questions –

show your working clearly

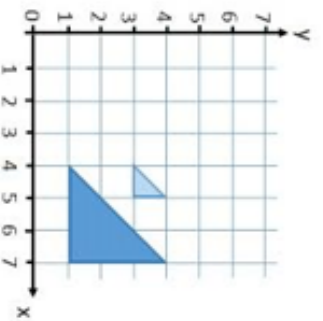
Fold over to check	Title	Formula	Write title	Check	Write Formula	Check
	Area of a triangle	$\frac{(b \times h)}{2}$				
	Area of a Trapezium	$\frac{1}{2}(a + b) \times h$				
	Area of a parallelogram	$b \times h$				
	Area of a circle	$\pi \times r^2$				
	Circumference of a circle	$\pi \times d$				
	Density	$\frac{mass}{volume}$				
	Pressure	$\frac{force}{area}$				
	Speed	$\frac{distance}{time}$				
	Pythagoras' Theorem	$a^2 + b^2 = c^2$				
	Sin (x)	$\frac{opp}{hyp}$				
	Cos (x)	$\frac{adj}{hyp}$				
	Tan (x)	$\frac{opp}{adj}$				
	Equation of a straight line	$y = mx + c$				
	Gradient	$m = \frac{rise}{run}$				
	y - intercept	+ c				
	Volume	Area of the cross-section x length				
	Number of sides	$\frac{360}{exterior\ angle}$				

1	Complete the following values without a calculator: 3.2×4.5 $5.2 \div 4$	5^2 2^a $\sqrt{16}$ $\sqrt[3]{1000}$ $5 + 3 \times 2$
2	 <p>Give the HCF and LCM of 420 and 132 HCF = LCM =</p>	
3	 <p>Write the elements of each set: $A \cup B$ $A \cap B$</p>	
4	Show $0 \leq x < 4$	
5	Identify each type of sample a) Each item has an equal chance of selection e.g. names out of a hat b) Selecting every "nth" item in a list c) Proportions in a sample are the same as the proportions in the population	a) b) c)
6	£500 in a bank. SIMPLE interest is added at 2% pa. for 3 years. Final amount?	
7	£800 in a bank. Compound interest is added at 1.5% pa. for 5 years. What would you put into calculator to find final amount?	
8	Dress costs £82 in 2019. Has decreased by 9%. What would you put into calculator to find price in 2018?	
9	What are the sum of the interior angles in a Pentagon?	
10	Regular exterior angle in a shape is 40° . Number of sides?	
11	Write the exact value of $\sin 30^\circ$	
12	Write the construction needed to show the locus of points equidistant from AB and BC	
13	Write the construction needed to show the locus of points equidistant from B and D	
14	Sketch each graph: a) $y = x$ b) $y = x^2$ c) $y = x^3$	

Describe the following single transformations:



15



16 A line has a gradient of 5 and goes through (2, 3). Equation?

16

17 Write the gradient of $y = \frac{2}{8}x + 7$

18 Write the y intercept of $y = -\frac{8}{7}x + 9$

19 Write the gradient of the line parallel to $y = -\frac{9}{11}x + 5$

20 Write the gradient of the line perpendicular to $y = \frac{8}{9}x - 7$

Identify the parallel vectors:

a) $5i - 3j$

b) $2i - j$

c) $15i - 9j$

d) $20i - 6j$

21

